Canonical form

Canonical form < Sum of minterms / Product of maxterms

Minterm: an AND term consists of all literals in their normal or complement form as its inputs

Ex) For two variables x and y, $(x'y', x'y, xy', xy)$ are minterms

* Also called a standard product.

Maxterm: an OR term consists of all literals $(x'+y', x'+y, x+y', x+y)$ are maxterms

* Also called a standard sum.

⇒ For n variables ⇒ 2^n minterms and 2^n maxterms

Table 2.3
Minterms and Maxterms for Three Binary Variables

x	y	z	Minterms Term	Minterms Designation	Maxterms Term	Maxterms Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

* $(m_i)' = M_i$ for $0 \le i \le 7$

Table 2.4
Functions of Three Variables

x	y	z	f_1'	Function f_1	m_1	m_4	m_7	Function f_2	
0	0	0	1	0	0	0	0	0	
0	0	1	0	1	1	0	0	0	
0	1	0	1	0	0	0	0	0	
0	1	1	1	0	0	0	0	1	m_3
1	0	0	0	1	0	1	0	0	
1	0	1	1	0	0	0	0	1	m_5
1	1	0	1	0	0	0	0	1	m_6
1	1	1	0	1	0	0	1	1	m_7

$m_1 \; m_4 \; m_7$

$x'y'z$

$f_1 = m_1 + m_4 + m_7$

$= x'y'z + xy'z' + xyz$

$f_2 = m_3 + m_5 + m_6 + m_7$

⇒ Sum of minterms

$f_1' = m_0 + m_2 + m_3 + m_5 + m_6$

$f_1 = (f_1')'$

$= (m_0 + m_2 + m_3 + m_5 + m_6)'$

$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$

⇒ Product of maxterms

Canonical form
(unique)

Algebraic expression → Canonical form

Given $F = A + B'C$

1° Sum of minterms

$$
\begin{aligned}
F &= A + B'C \\
&= A\cdot(B+B') + B'C \\
&= AB + AB' + B'C \\
&= AB\cdot(C+C') + AB'(C+C') + (A+A')B'C \\
&= ABC + ABC' + AB'C + AB'C' + A'B'C \\
&= m_7 + m_6 + m_5 + m_4 + m_1
\end{aligned}
$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F = \Sigma m(1,4,5,6,7) = \Sigma(1,4,5,6,7)$

2° Product of maxterms

$$F = A + B'C$$
$$= (A+B')\cdot(A+C)$$
$$= (A+B'+C\cdot C')\cdot(A+BB'+C)$$
$$= (A+B'+C)(A+B'+C')\cdot(A+B+C)$$
$$= M_2 M_3 M_0$$

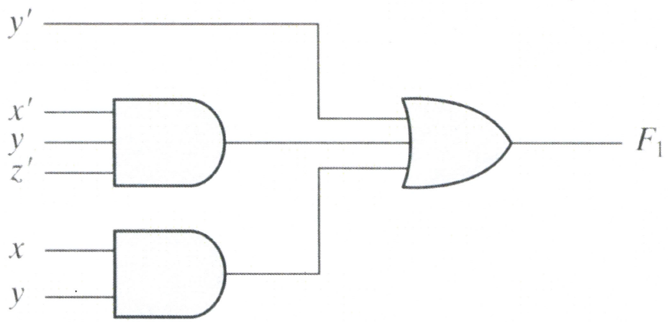
or $F' = m_0 + m_2 + m_3$

$$\therefore F = (m_0 + m_2 + m_3)' = M_0 M_2 M_3$$
$$= \prod M(0,2,3) \text{ or } \prod(0,2,3)$$

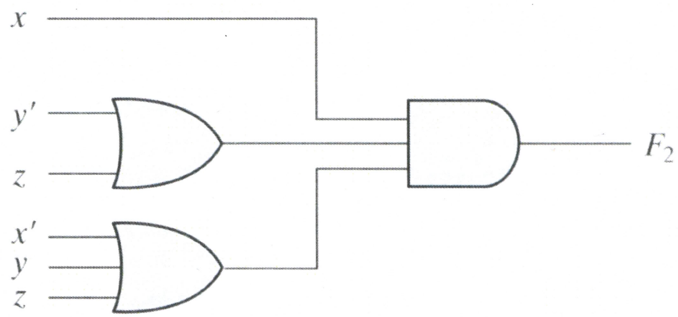
A	B	C	F	OR		
0	0	0	0	0	$\not0$ 1	1
0	0	1	1	1	1	1
0	1	0	0	1	0	1
0	1	1	0	1	1	0
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1
				M_0	M_2	M_3

Standard form

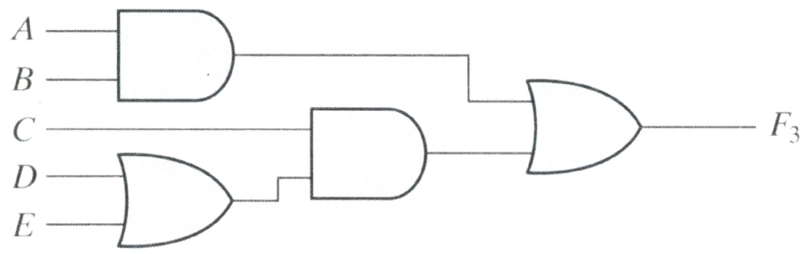
Sum of products / product of sums } two-level implementation



(a) Sum of Products $F_1 = y' + x'yz' + xy$

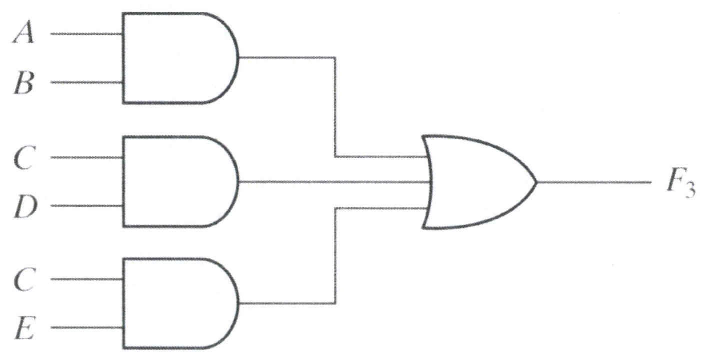


(b) Product of Sums $F_2 = x \cdot (y'+z) \cdot (x'+y+z)$



(a) $AB + C(D + E)$

three-level



(b) $AB + CD + CE$ ⇒ less delay

two-level