高等磁共振影像技術

動態加速影像與壓縮感知
Accelerated MRI & Compressed Sensing

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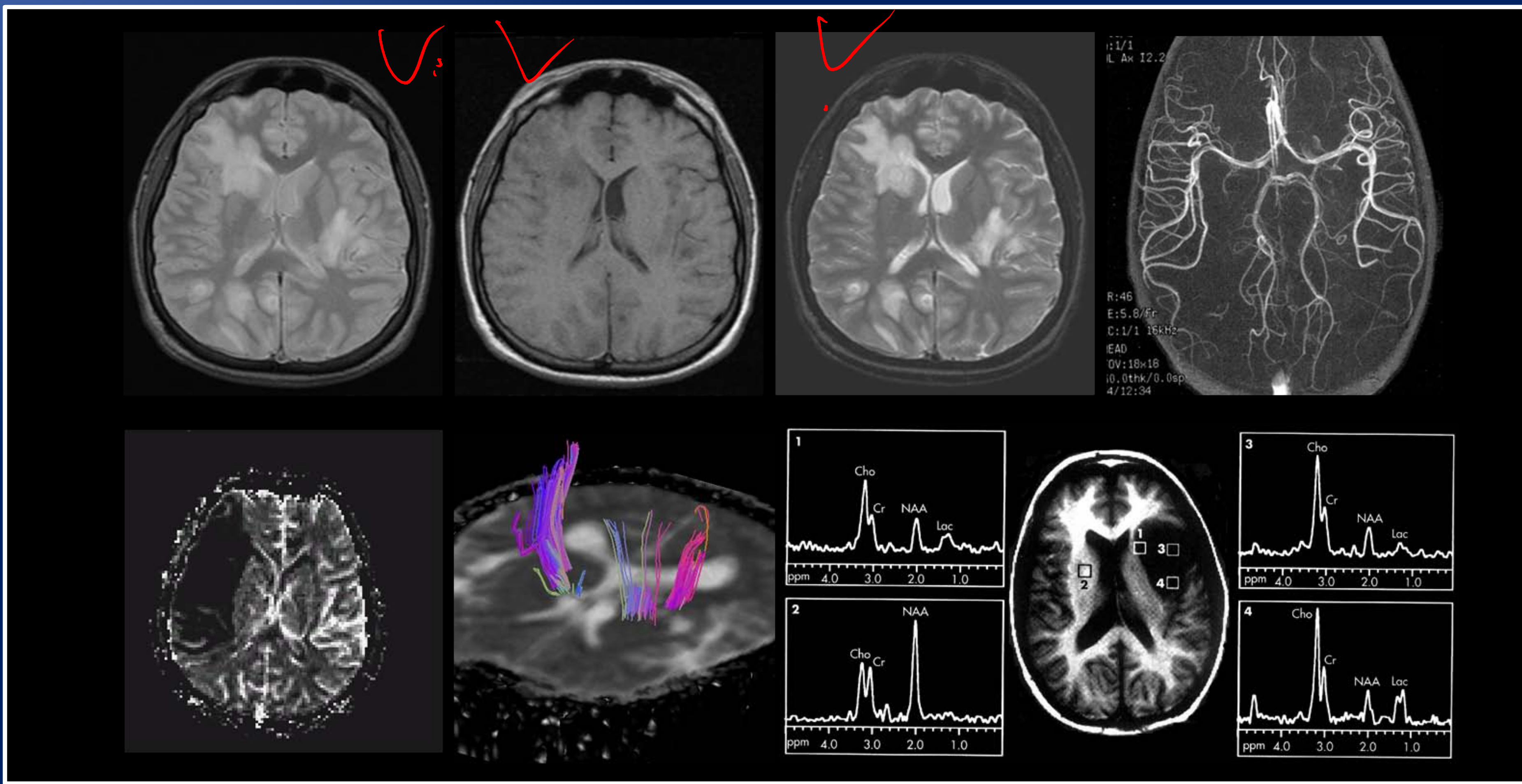
Accelerated MRI & Compressed Sensing

- Time requisite in MRI
  - One k-space line at a time
  - Several minutes for one volumetric image

- Acceleration
  - Physical limit : Contrast must be preserved
  - Hardware limit : Gradient Performances
  - Software limit : Nyquist Criteria

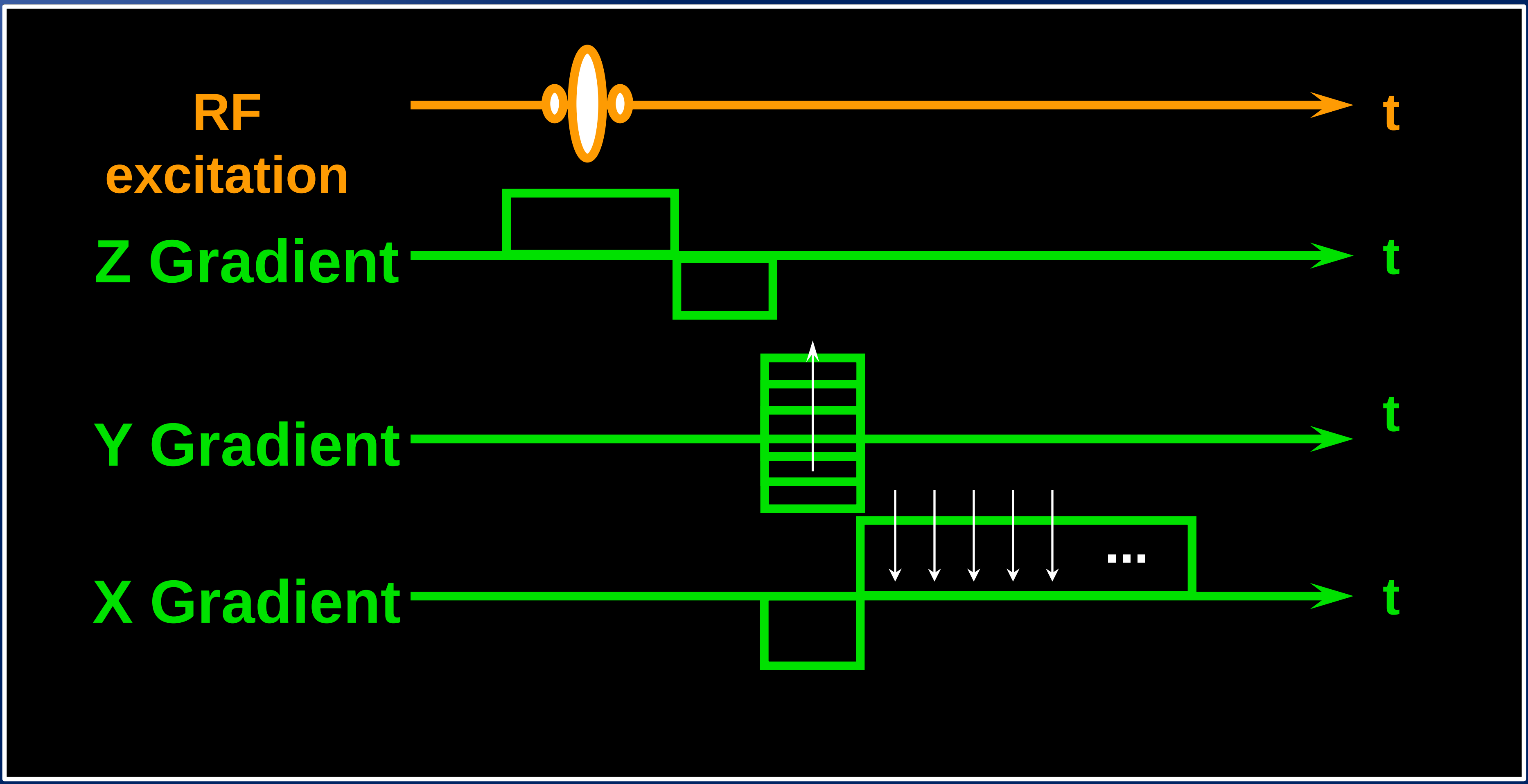
Physical Limit



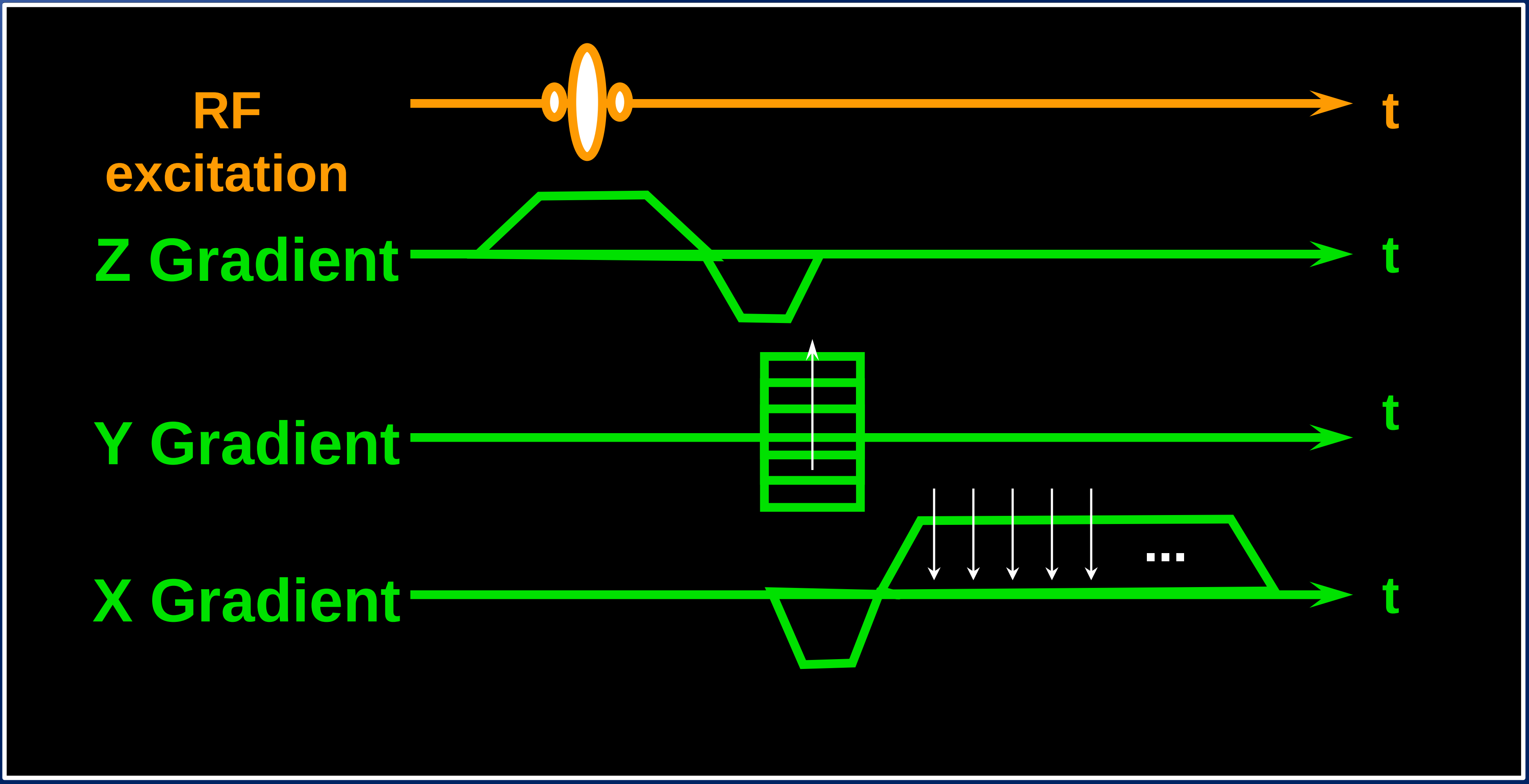
Physical Limit

- Signal feature ~ Repetition Time
- Getting Faster = Loss in Contrast
- Magnetization Preparation?!
  - – EPI : Distortion
  - – GRE/FSE: Signal Inhomogeneity

Hardware Limit: IDEAL



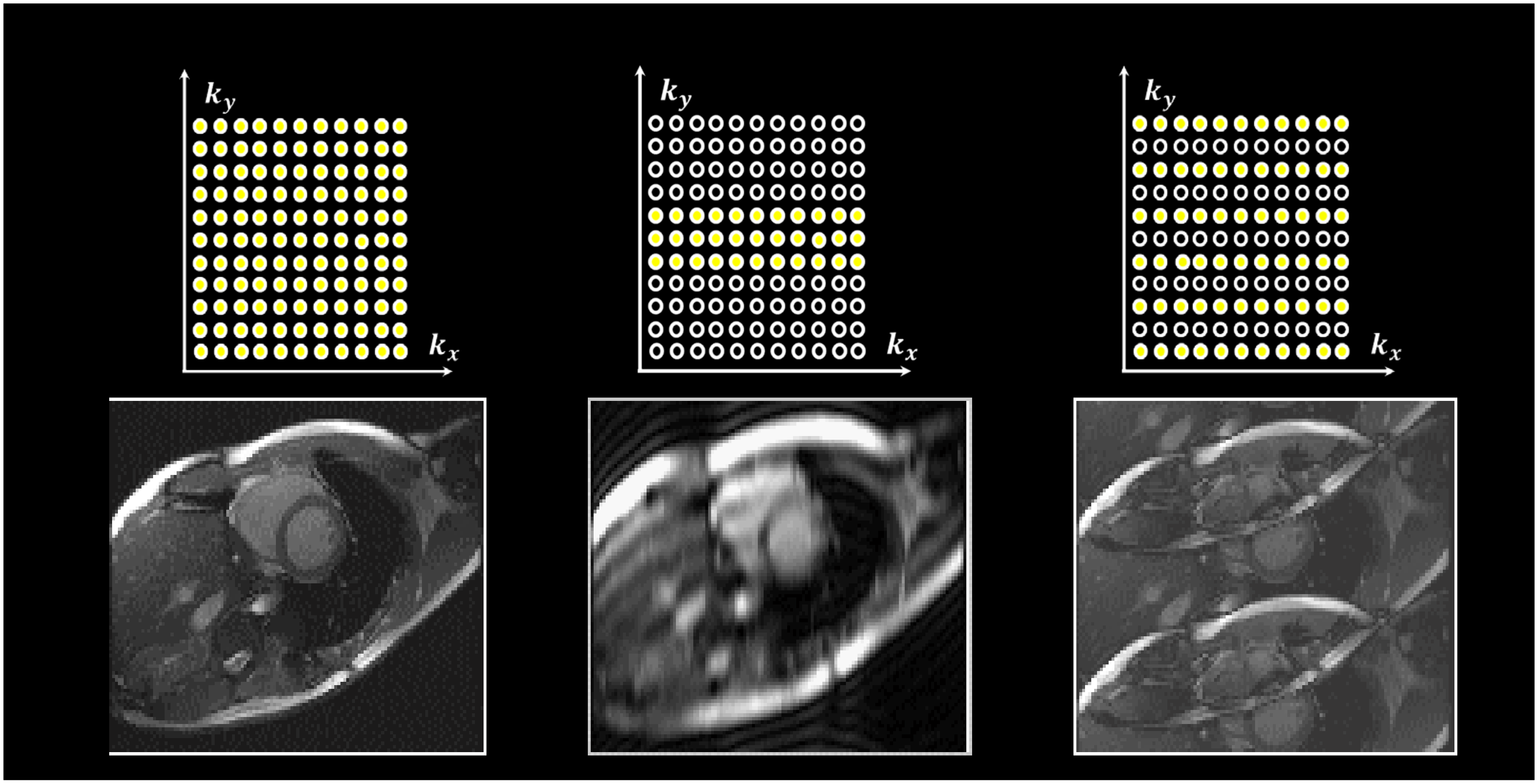
Hardware Limit



Hardware Limit

- Higher Slew Rate:
  - Shorten the ramp
  - Require better eddy current shielding
  - Peripheral Nervous Stimulation
- $ is also a kind of hardware

Software Limit



Software Limit

- Fourier Encoding and reconstruction
- Imaging Speed v.s. Resolution
- Nyquist Criteria
  - Linear perspective:
    The number of conditions should be compatible to the number of variables

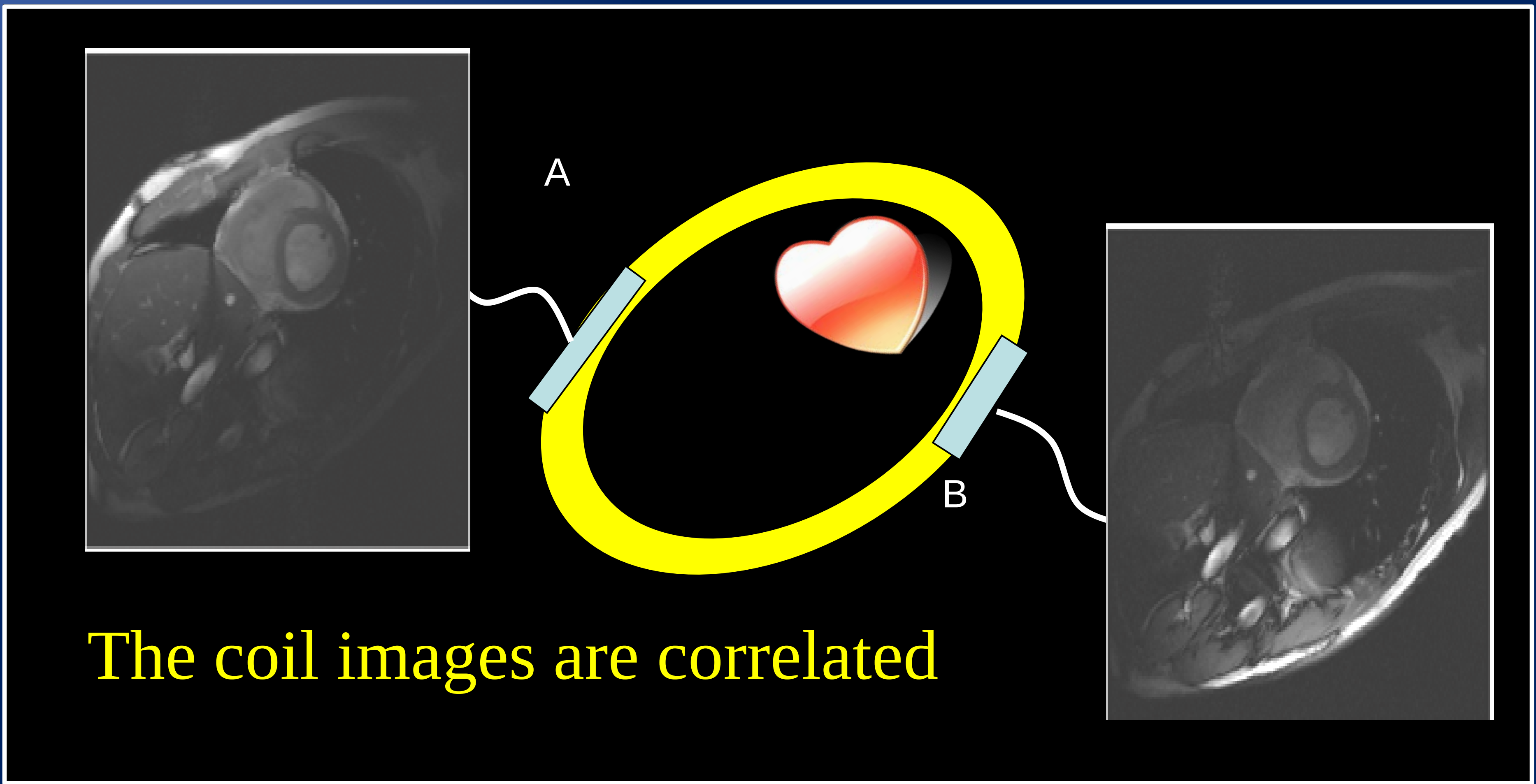
Software Limit

- Fourier Encoding
- $k(k_x) = \sum_{x=0}^{N_x-1} \rho(x) e^{-2\pi i \frac{k_x x}{N_x}}$
- $\begin{bmatrix} k(0) \\ \vdots \\ k(N_x - 1) \end{bmatrix} = \begin{bmatrix} e^{-2\pi i \frac{(0*0)}{N_x}} & \dots & e^{-2\pi i \frac{(0*N_x-1)}{N_x}} \\ \vdots & \ddots & \vdots \\ e^{-2\pi i \frac{(N_x-1*0)}{N_x}} & \dots & e^{-2\pi i \frac{(N_x-1*N_x-1\ )}{N_x}} \end{bmatrix} \begin{bmatrix} \rho(0) \\ \vdots \\ \rho(N_x - 1) \end{bmatrix}$

Accelerated MRI

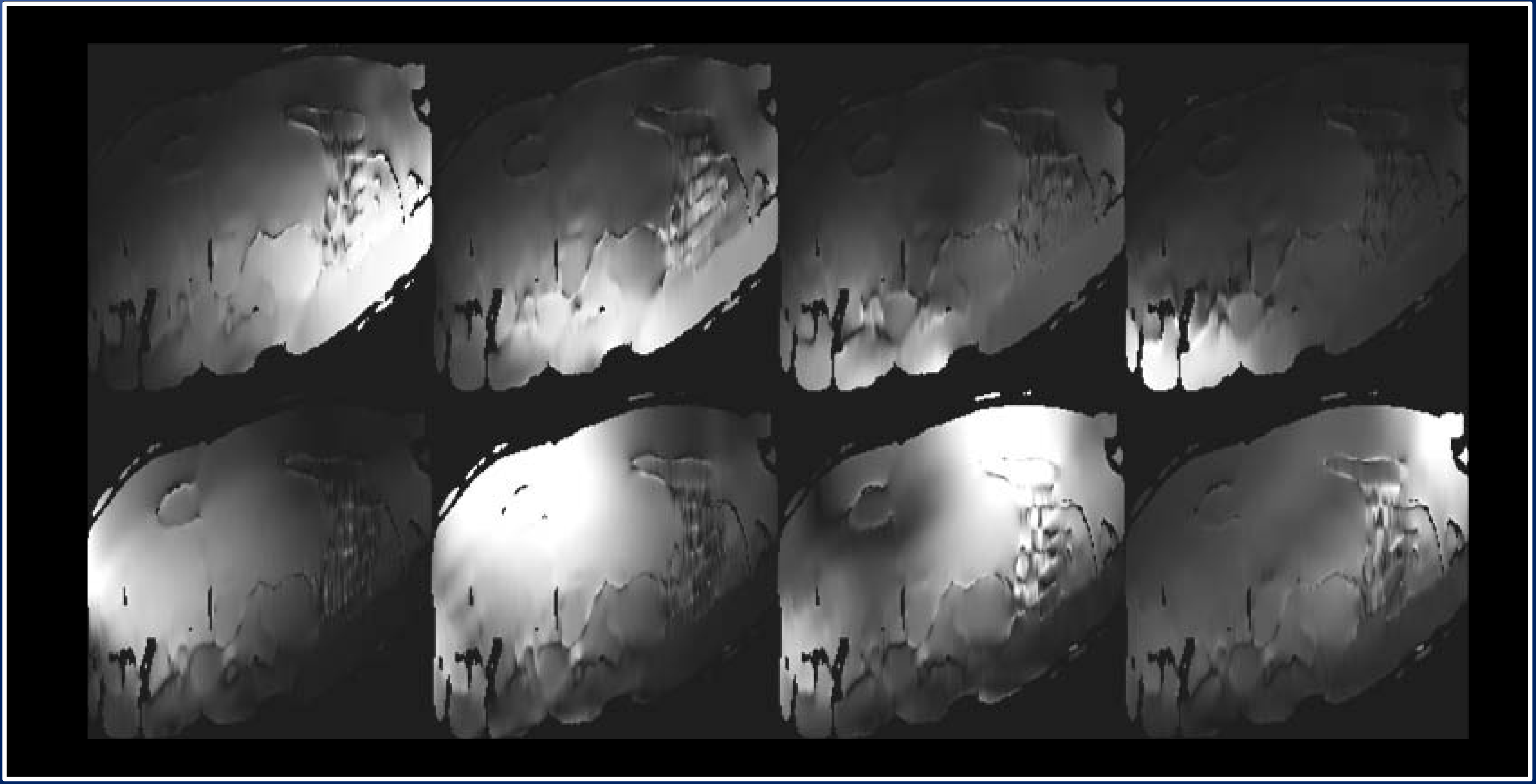
- Hardware + Software
  - Parallel Imaging : PILS, SMASH, GRAPPA, SENSE, Space-RIP …..
- Temporal Strategies
  - UNFOLD, kt-BLAST, TSENSE,…..
- Compressed Sensing

Parallel Imaging

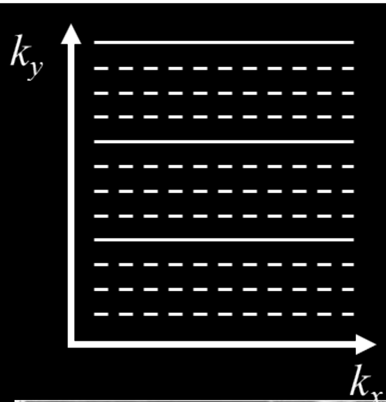


The coil images are correlated

Parallel Imaging - SENSE



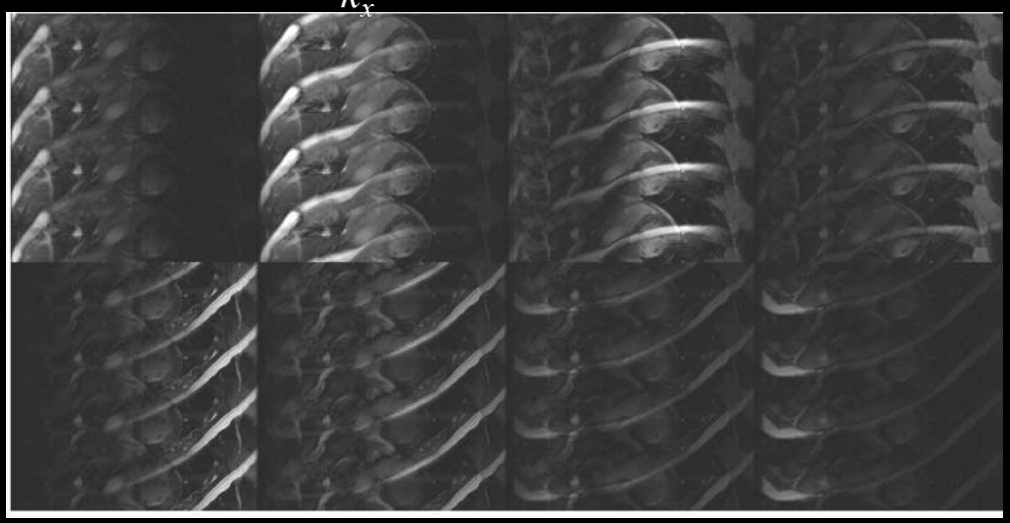
Parallel Imaging

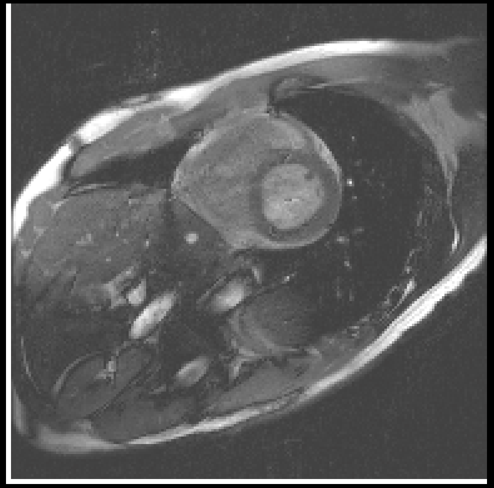


$$\begin{bmatrix} \tilde{X}_1(x,y_1) \\ \tilde{X}_2(x,y_1) \\ \vdots \end{bmatrix} = \begin{bmatrix} S_1(x,y_1) & S_1(x,y_2) & \dots \\ S_2(x,y_1) & S_2(x,y_2) & \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} X(x,y_1) \\ X(x,y_2) \\ \vdots \end{bmatrix}$$

$$\tilde{\mathbf{X}} = \mathbf{S}\mathbf{X}$$

$$\mathbf{X} = (\mathbf{S}^{\mathrm{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\tilde{\mathbf{X}}$$





Parallel Imaging

- The signal equation

$$\boldsymbol{k}_{\vec{\mathbf{k}}} = \mathbf{P}_{\vec{\mathbf{k}}}\, \mathbf{F}_{\vec{\mathbf{r}}\rightarrow\vec{\mathbf{k}}}\, \mathbf{S}_{\vec{\mathbf{r}}}\boldsymbol{\rho}_{\vec{\mathbf{r}}} = \boldsymbol{\Phi}\boldsymbol{\rho}_{\vec{\boldsymbol{r}}}$$

- r: image space, k: k-space
- k : k-space signal P : sampling mask

  F : Fourier Operator S : Sensitivity Encoding
- Φ : a general encoding matrix

General Solution of Parallel Imaging

$$\rho'_{\vec{\mathrm{r}}} = \left(\mathrm{P}_{\vec{\mathrm{k}}}\,\mathrm{F}_{\vec{\mathrm{r}}\to\vec{\mathrm{k}}}\,\mathrm{S}_{\vec{\mathrm{r}}}\right)^{-1} k_{\vec{\mathrm{k}}} = \left(\Phi^{\dagger}\Phi\right)^{-1}\Phi^{\dagger} k_{\vec{\mathrm{k}}}$$

Or

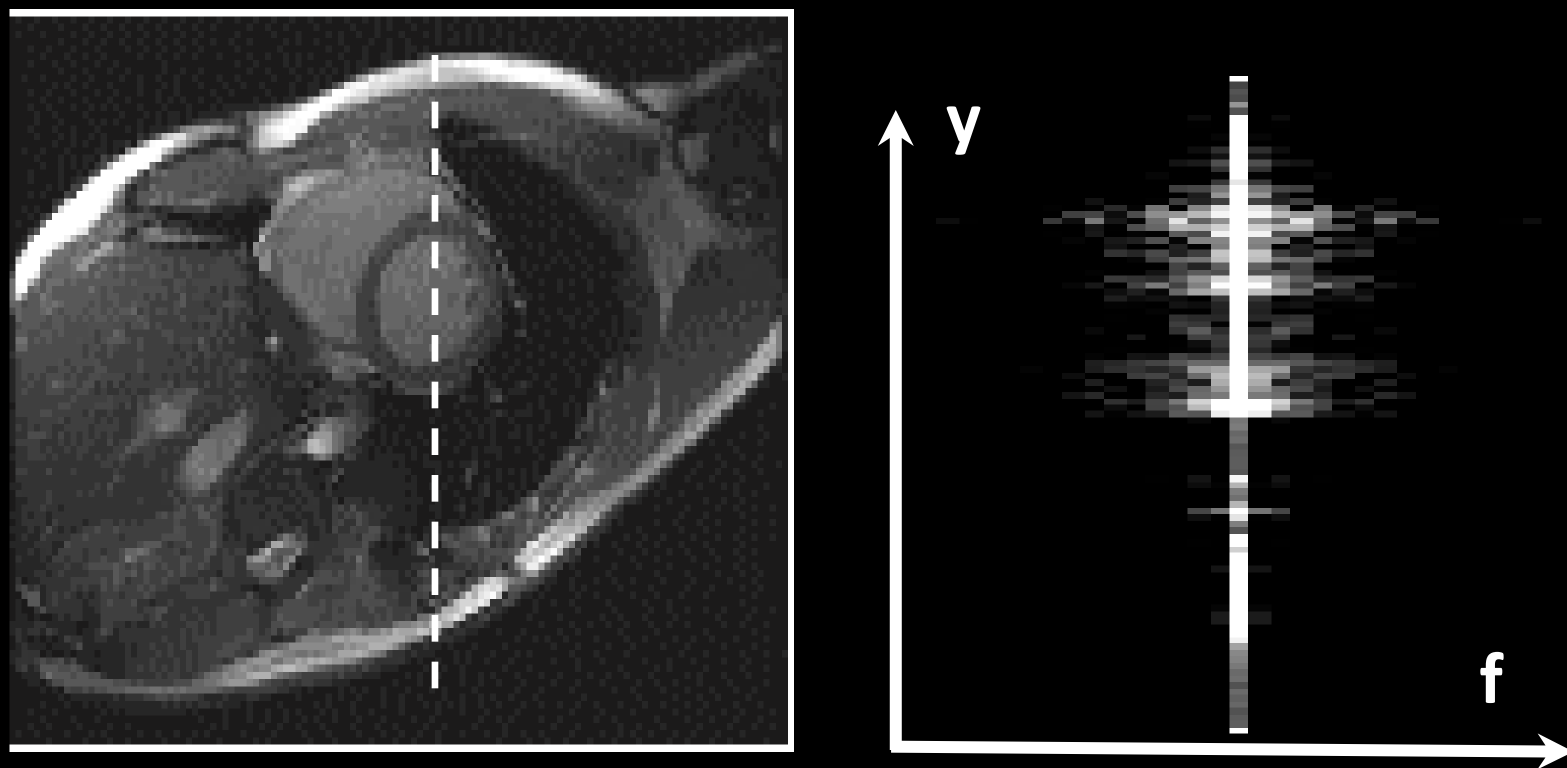
$$\rho'_{\vec{\mathrm{r}}} = min_{\rho'_{\vec{\mathrm{r}}}} \left|k_{\vec{\mathrm{k}}} - \mathrm{P}_{\vec{\mathrm{k}}}\,\mathrm{F}_{\vec{\mathrm{r}}\to\vec{\mathrm{k}}}\,\mathrm{S}_{\vec{\mathrm{r}}}\rho'_{\vec{\mathrm{r}}}\right|_2^2$$
$$= min_{\rho'} \left|k - \Phi\rho'\right|_2^2$$

- Parallel imaging still satisfies Nyquist criteria.
- Sensitivity encoding serves the additional conditions

Accelerated MRI

- Hardware + Software
  - Parallel Imaging : PILS, SMASH, GRAPPA, SENSE, Space-RIP …..
- Temporal Strategies
  - UNFOLD, kt-BLAST, TSENSE,…..
- Compressed Sensing

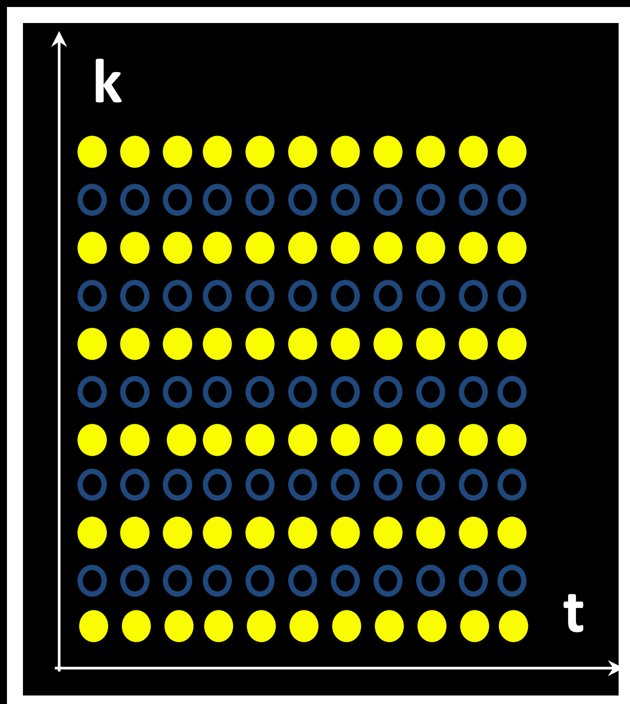
The y-f power spectrum of a heart



Cardiac CINE

y-f power spectrum

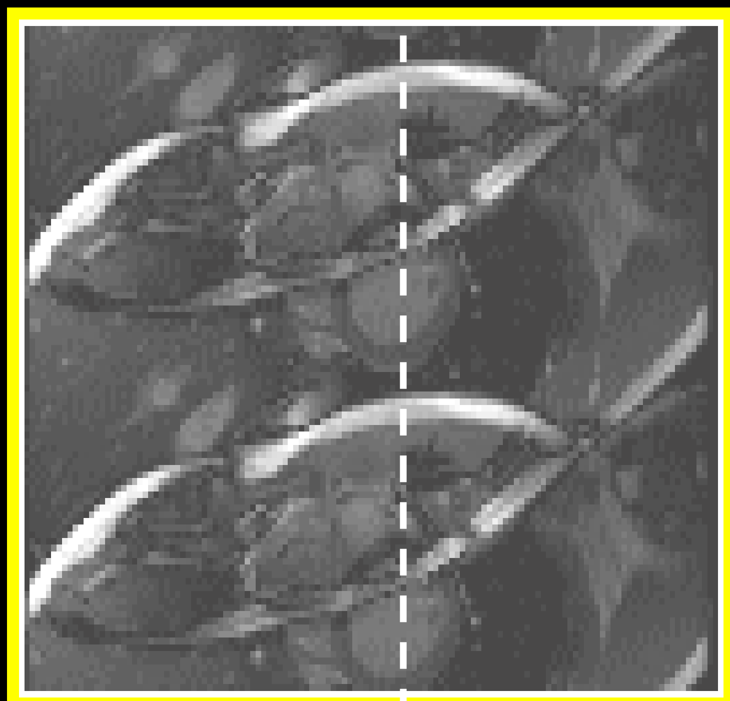
Acceleration Factor R = 2



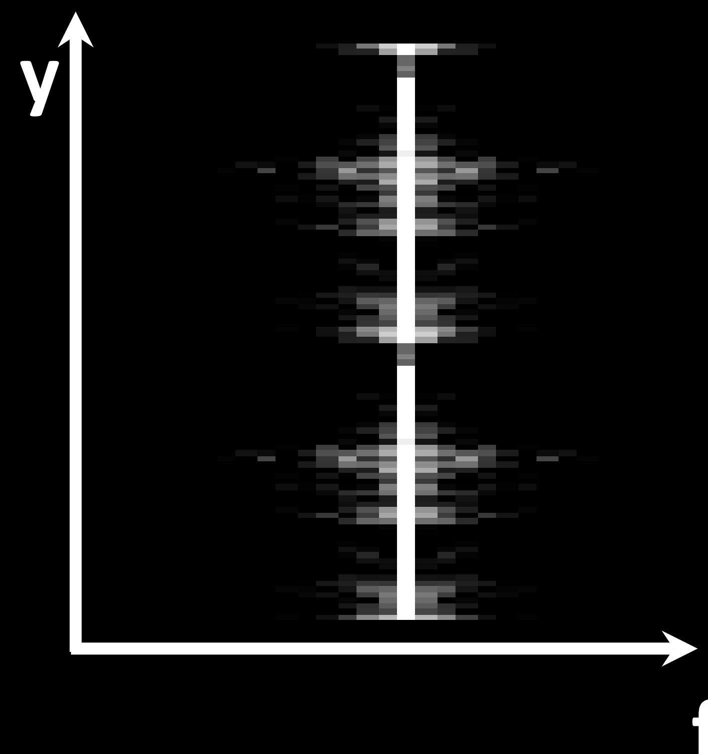
- kt space sampling

R = 2 Image

Impact on the y-f power spectrum

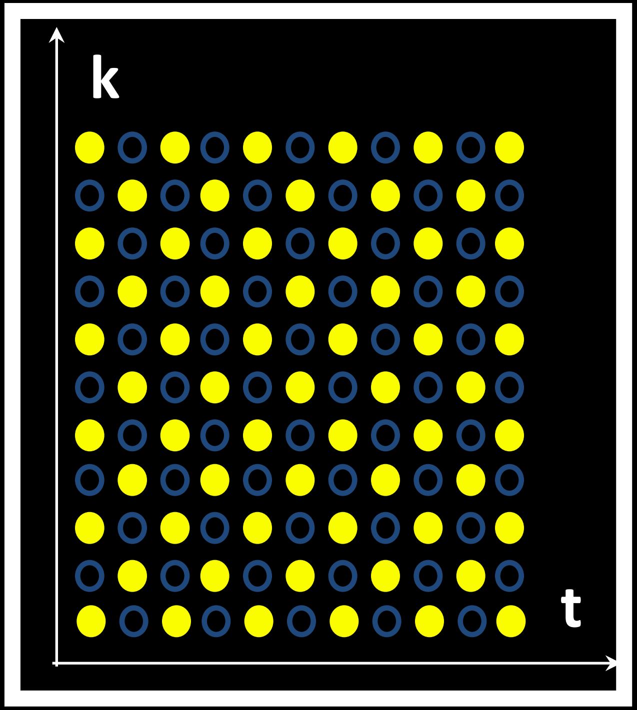


- R = 2 Image



y-f power spectrum

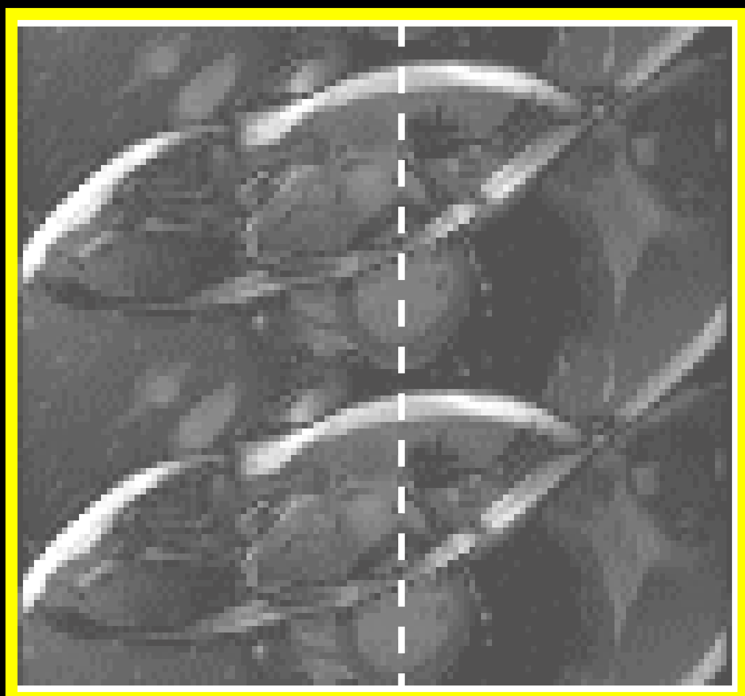
Temporal Strategy R = 2

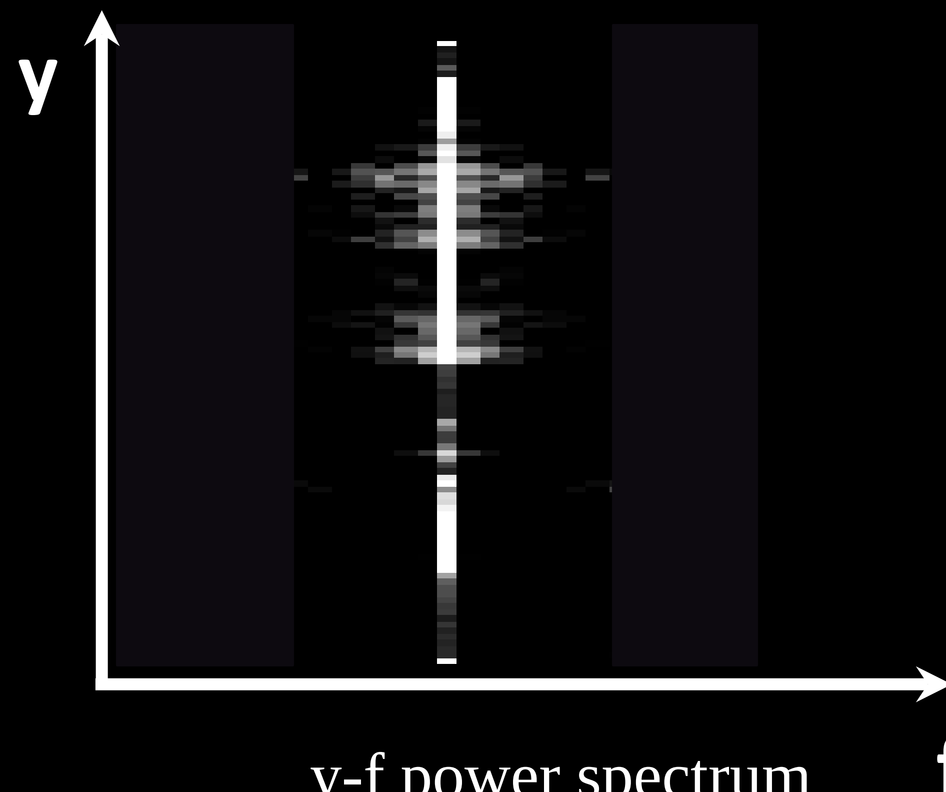


- kt space sampling

R = 2 Image

Undersampling 的變化 (以兩倍為例)

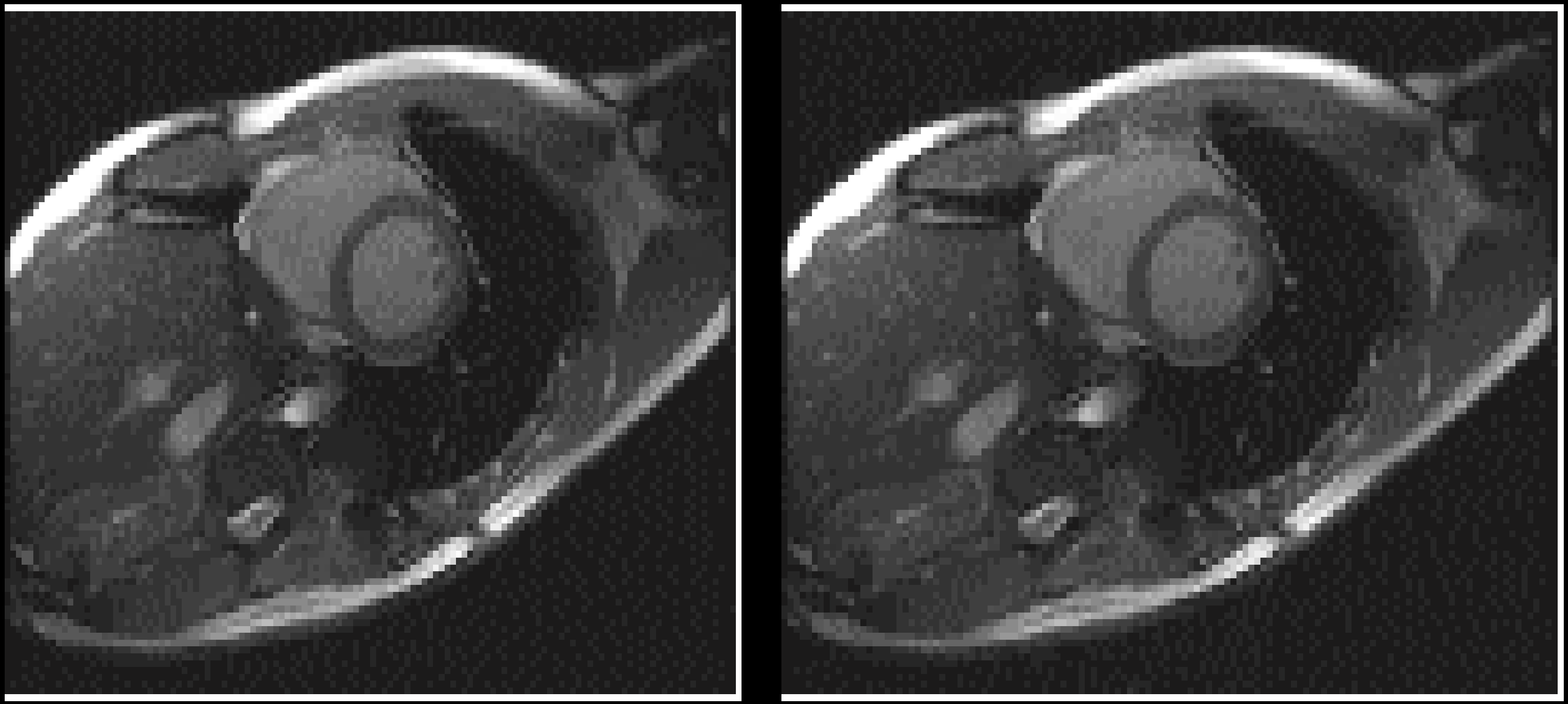




- R = 2 Image

y-f power spectrum

UNFOLD



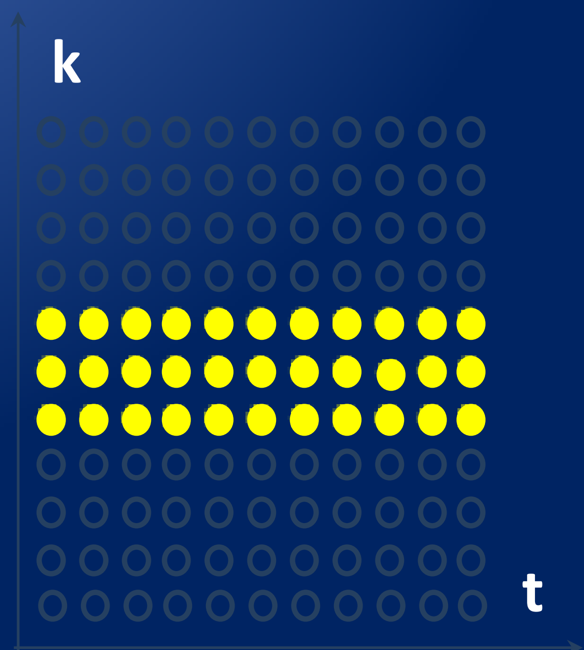
- Original

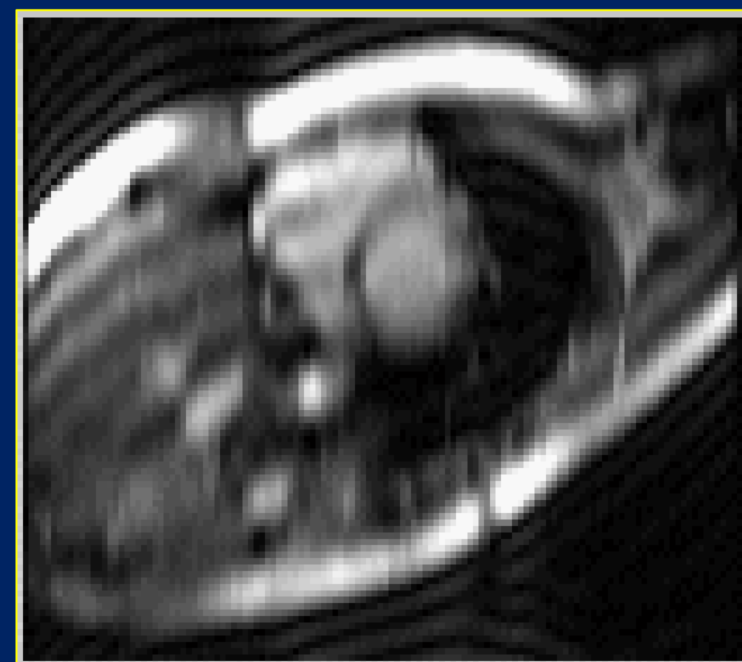
UNFOLD

Temporal Undersampling Strategy

- Aliasing artifact can be removed by filtering
- UNFOLD (by Bruno Madore)
- Extended research topics
  - kt-BLAST, TSENSE
  - Compressed Sensing

Undersampling 的變化 (只取低頻)

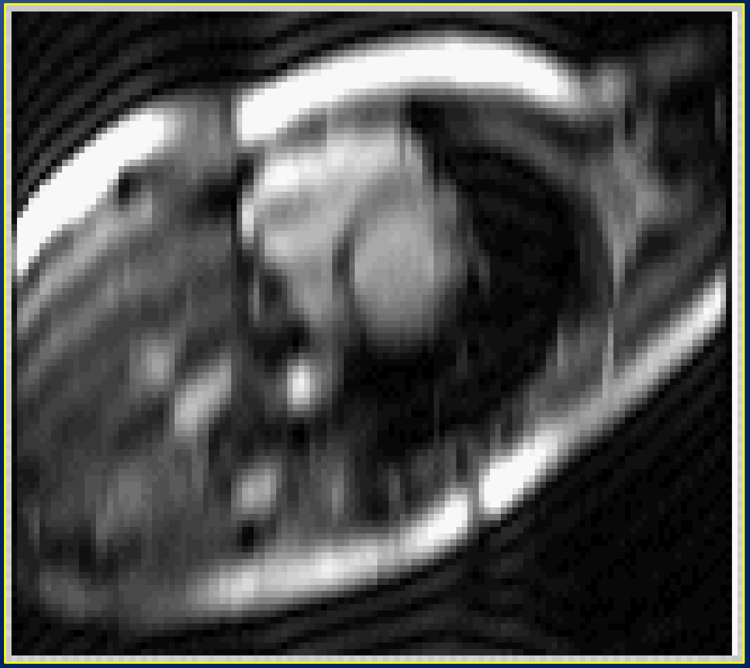


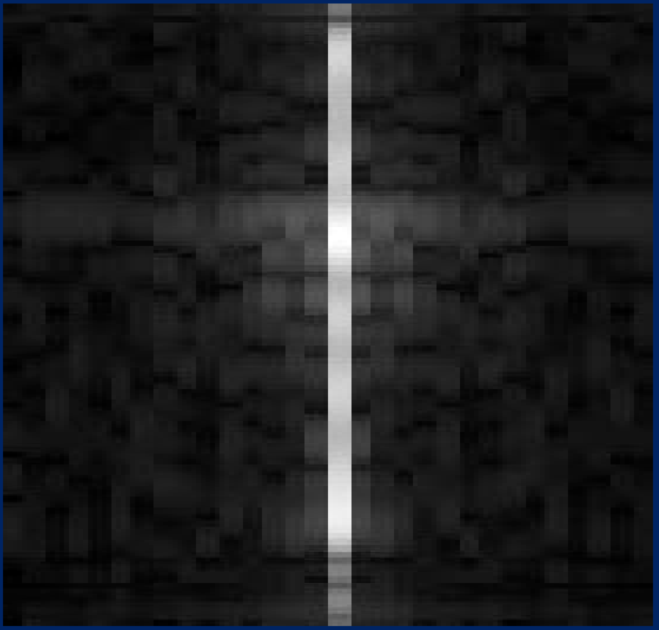


- kt space sampling

Low Res Image

The spectrum of the low-res images

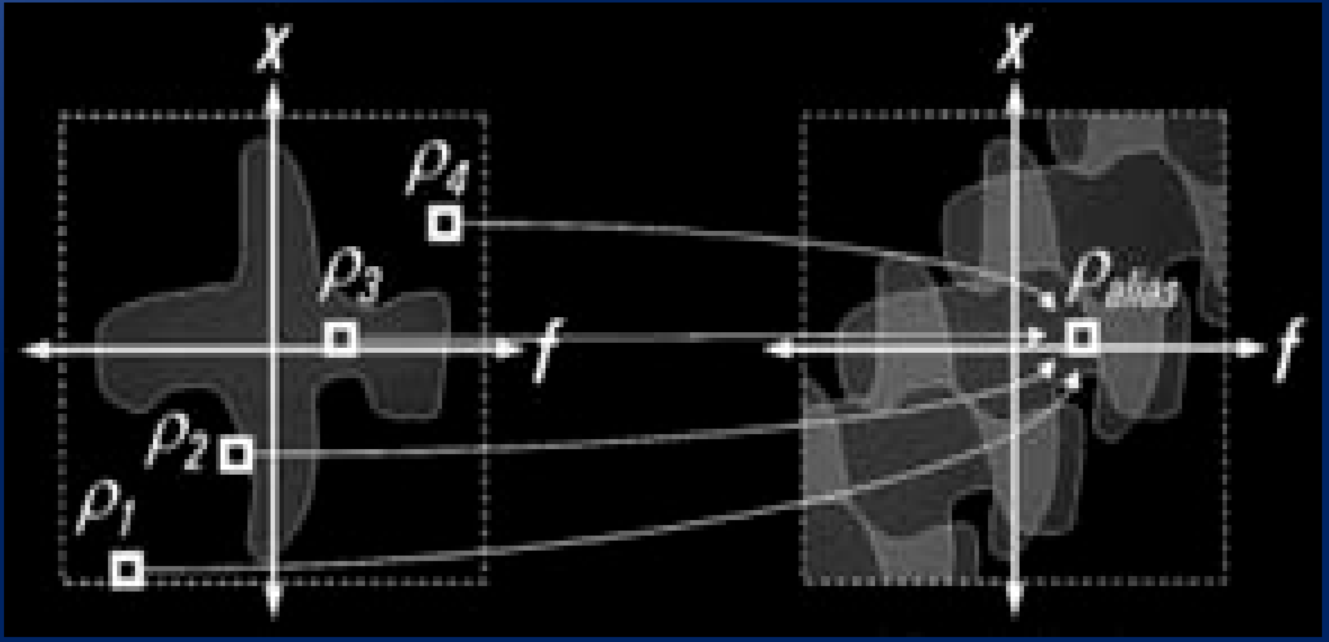




- Low Res Image

yf space pattern

kt-BLAST、kt-SENSE



由低解析度影像，取得訊號權重資訊

經由訊號權重，解開aliased signal

只要能解開 aliasing，影像重建就不是問題

Temporal Strategy & Reconstruction

- Aliasing artifact can be relocated along f domain by special designed sampling pattern.

- The reconstruction requires prior knowledge of the signal behavior
  - UNFOLD: No signal appears in Nyquist region
  - kt-BLAST: a low resolution prior knowledge is required for the reconstruction.

Reconstruction Algorithm

- Encoding Process

$$\boldsymbol{k} = \mathbf{PFS}\boldsymbol{\rho} = \boldsymbol{\Phi}\boldsymbol{\rho}$$

- Image Reconstruction Process

$$\boldsymbol{\rho}' = \min_{\boldsymbol{\rho}'} |\boldsymbol{k} - \mathbf{H}\boldsymbol{\Phi}\boldsymbol{\rho}'|_2^2 - \lambda^2 |\mathbf{L}\boldsymbol{\rho}|_2^2$$

$$\boldsymbol{\rho}' = \left(\boldsymbol{\Phi}^\dagger \mathbf{H}^\dagger \mathbf{H}\boldsymbol{\Phi} + \lambda^2 \mathbf{L}^\dagger \mathbf{L}\right)^{-1} \boldsymbol{\Phi}^\dagger \boldsymbol{k}$$

H: Implicit Regularization (FILTER)

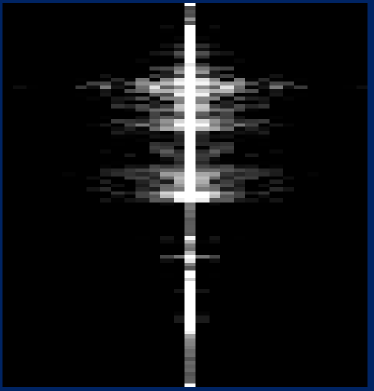
L: Explicit Regularization (FILTER)

Accelerated MRI

- Hardware + Software
  - Parallel Imaging : PILS, SMASH, GRAPPA, SENSE, Space-RIP …..
- Temporal Strategies
  - UNFOLD, kt-BLAST, TSENSE,…..
- Compressed Sensing

Compressed Sensing

- Much empty space in the power spectrum
  - A lot of empty space (0) : ***Sparse representation***
  - No. of SIGNIFICANT variables are much smaller than expected.



- Are there other sparse presentations?
  - For other images
- Is it possible to sample these significant components DIRECTLY?
  - Compressive Sampling

Compressed Sensing

- Sparse Representation
- Compressive Sampling
- Signal Recovery

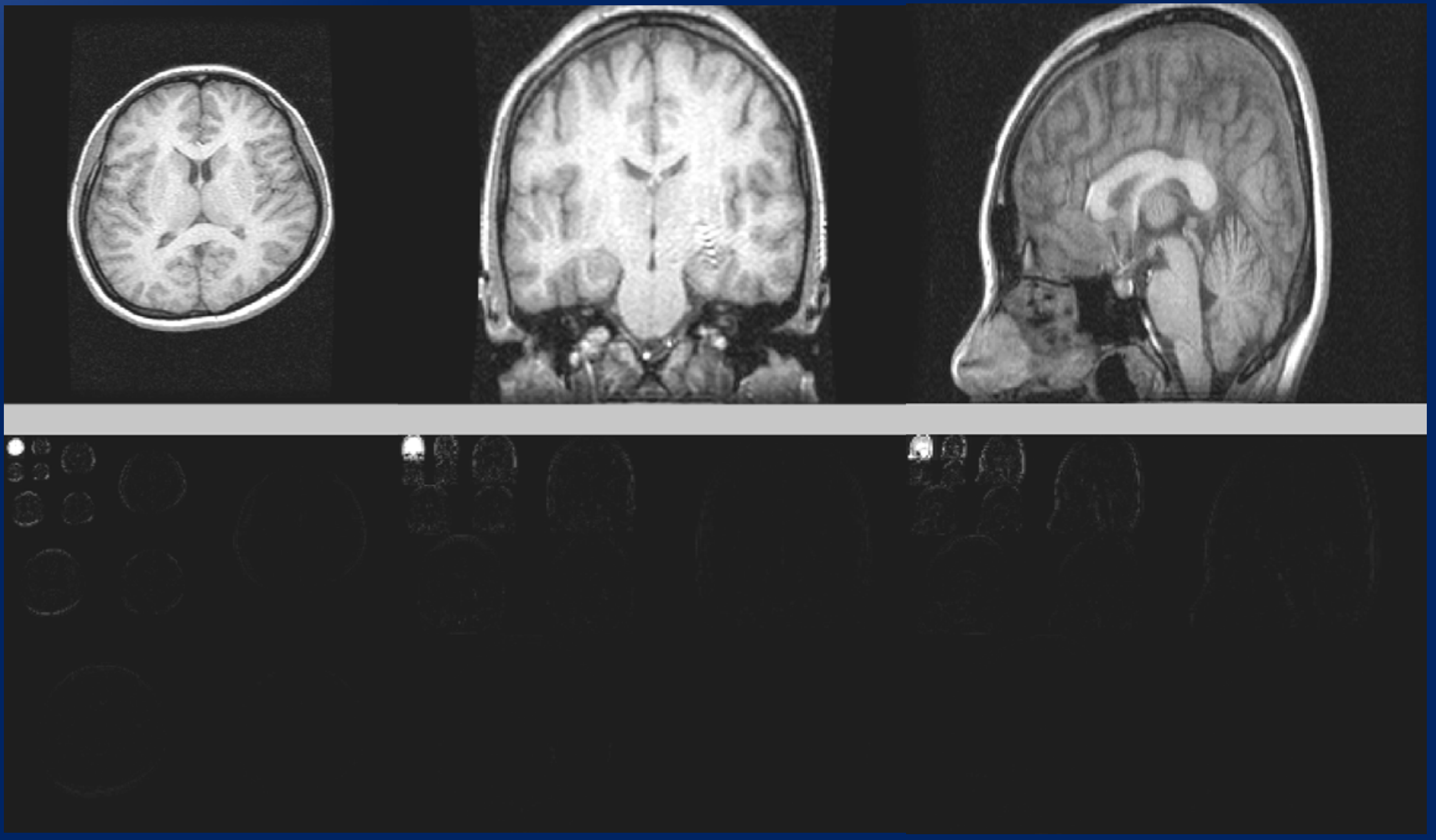
Compressed Sensing

- Sparse Representation
- Compressive Sampling
- Signal Recovery

Other Sparse Representation Wavelet Transform

- Data can sparsely represented

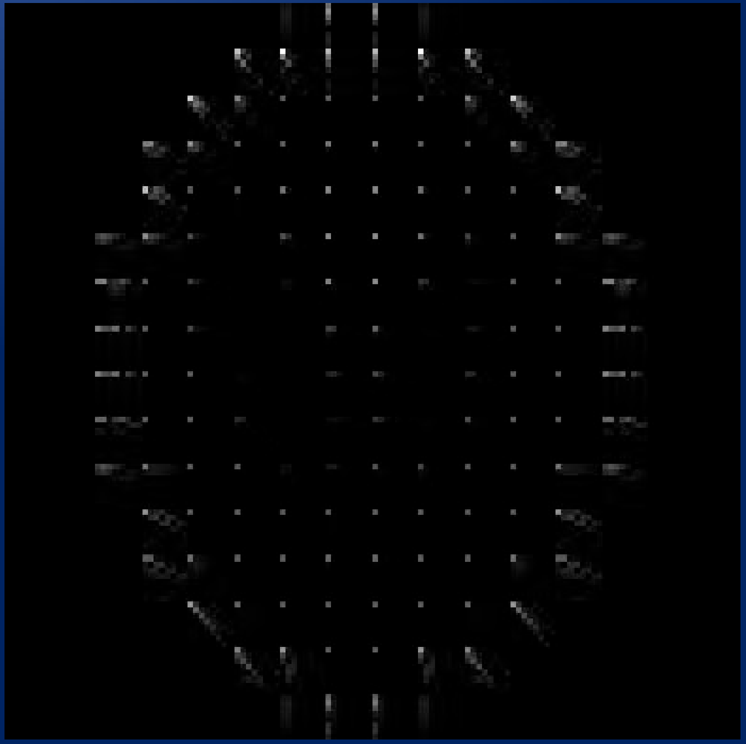
3D T1w



Wavelet Representation

Other Sparse Representation

Discrete Cosine Transform



Total Variance (Gradient)

Sparse transformations

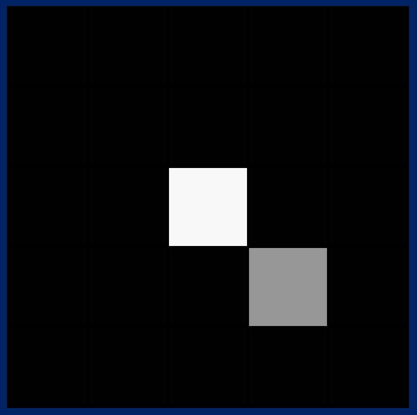
- Fourier transformation
- Wavelet transformation
- Discrete Cosine transformation
- Principal component decomposition
- Edge detection (Total Variance)

Data Compression & Sparsity

- Sparse Data can be compressed

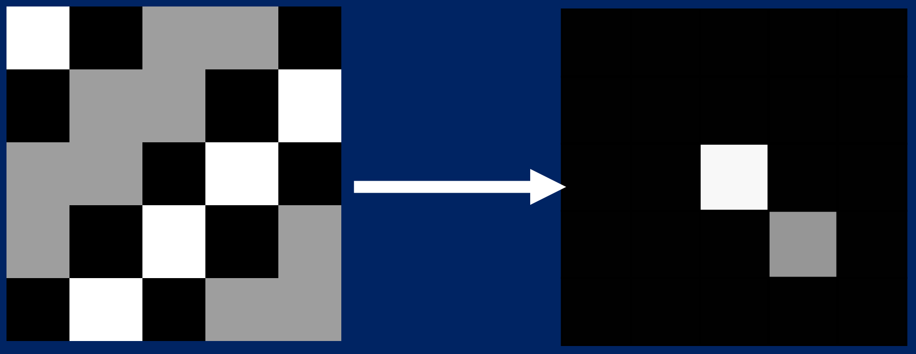
- Sparse Transformation
  - Fourier Transform
  - Wavelet (JPEG2000)
  - Discrete Cosine Transform (JPEG)
  - …etc

- Sparse Data



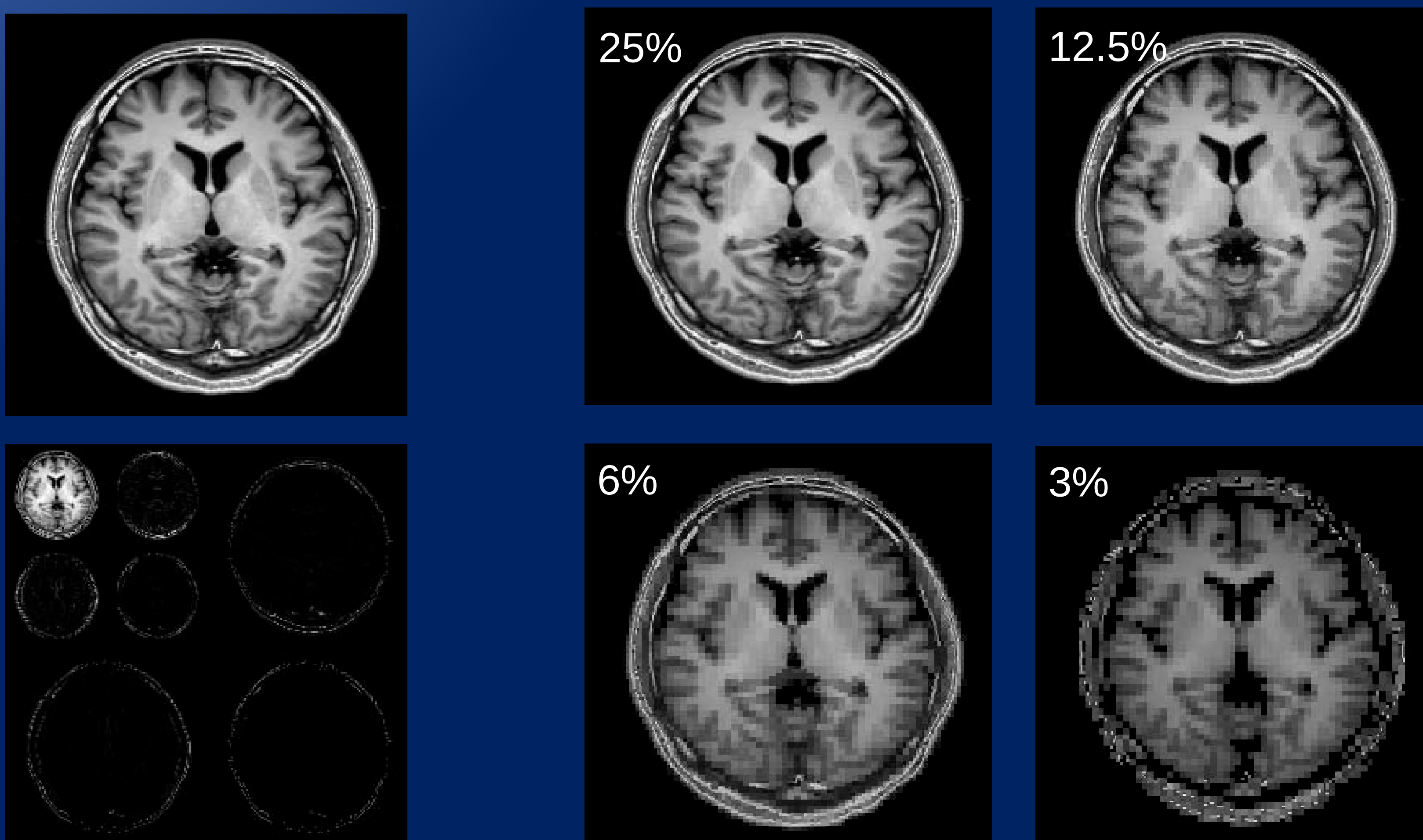
(13,64) (18,32)

- Transformed Sparse Data



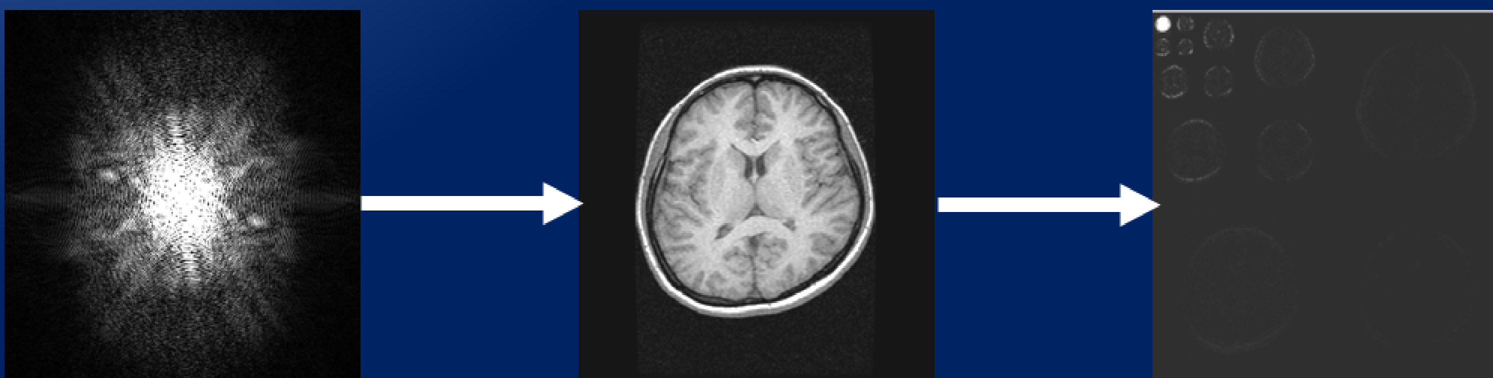
Other Sparse Representation

- Sparse data can be compressed



Where Compressed Sensing is from

- Something is wrong with "Sample then Compress"



~20%
data
stored
256x25

- What if a directly sampling from compressed data is feasible?

Compressed Sensing

- Sparse Representation
- Compressive Sampling
- Signal Recovery

Sampling equation (Ill-posed)

- The signal equation

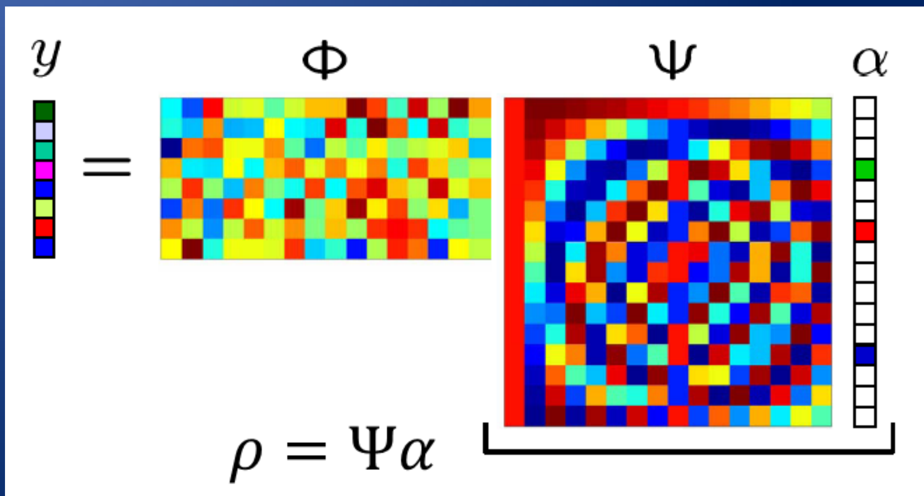
$$\boldsymbol{k} \quad = \mathbf{P}\,\mathbf{F}\,\boldsymbol{\Psi\alpha} = \boldsymbol{\Phi\Psi\alpha}$$

Ψ : Sparse Transformation s.t. $\Psi^{-1}\rho = \alpha$ is a sparse signal

F : A complete linear transformation

P : Sampling function on F space

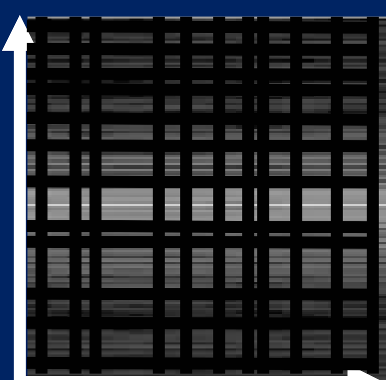
Sampling equation (Ill-posed)



$$\mathbf{k} \quad = \Phi\Psi\alpha$$

$$\rho: y - f \ signal$$

$$\Psi: \mathrm{I}, \Phi: \text{masked 2D FT}$$



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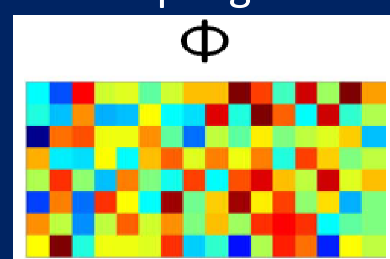
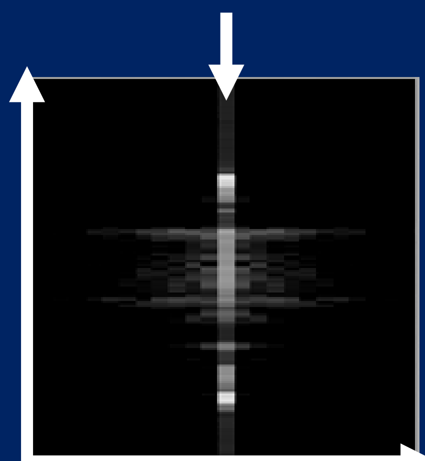
Compressive Sampling

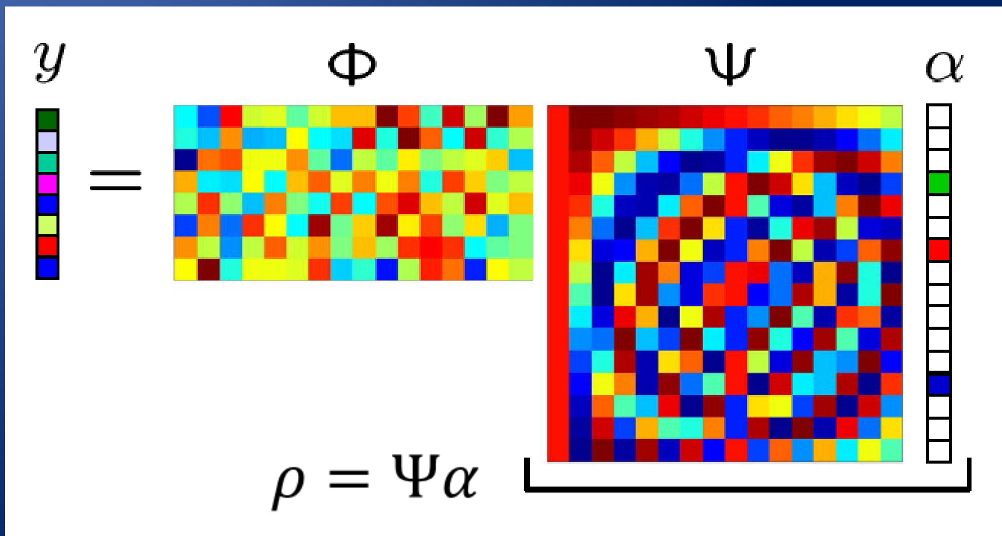


?????



CS Tutorial by Roemburg et al.

Sampling equation (Ill-posed)



$$y = \Phi\rho = \Phi\Psi\alpha$$

$\rho : object\,(N \times 1)$

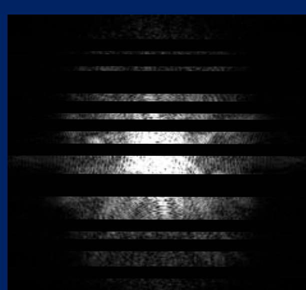
$\alpha : K - sparse\ data\,(N \times 1)$

$\Psi : Sparse\ Transform\,(N \times N)$

$\Phi : Encoding\ function\,(M \times N)$

$y : acquired\ data\,(M \times 1)$

$K < M \leq N$



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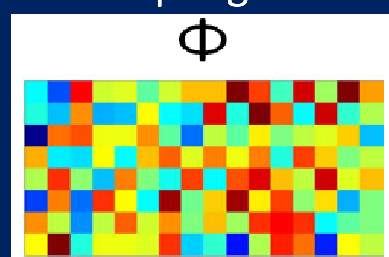
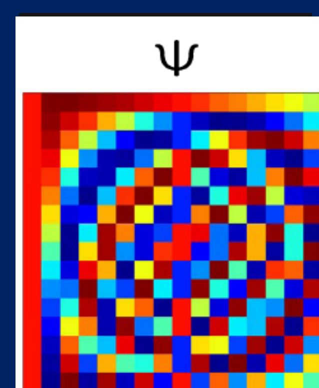
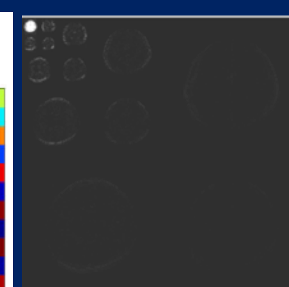
Compressive Sampling



?????

Wavelet





CS Tutorial by Roemburg et al.

Compressive Sampling

- Sparse Representation
- Incoherent sampling & representation space
- Restricted Isometry Property

Sparsity

- Sparse : only a few non-zero elements.
- Lp-norm $0 \leqq p<2$ can be used to represent the sparsity of a dataset. L0 and L1 are commonly used for discussion.

$$\|\vec{a}\|_p = \sqrt[p]{\sum_i |a_i|^p}$$

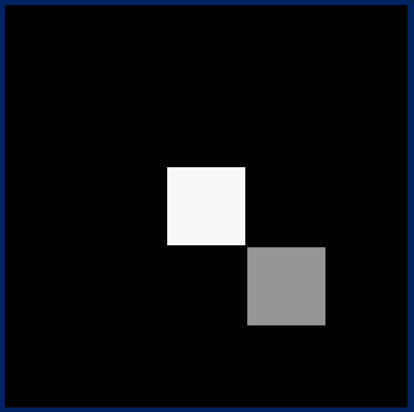
  - Ex: a=[5, 0, 0, -1, 0], $\|a\|_0=2$, $\|a\|_1=6$, precisely sparse
    b=[5, -4, 2, 3, 0], $\|b\|_0=4$, $\|b\|_1=14$, not sparse
    c=[-5, 0.001,-0.005, 1, 0],
    $\|c\|_0=4$, $\|c\|_1=6.006$, nearly sparse (with noise and …)
- In Compressed Sensing
  - L1 norm is suitable for the nearly sparse data.
  - L1 is equivalent to L0 in reconstruction. [Donoho, Tanner]

Data Compression & Sparsity

- Sparse Data can be compressed

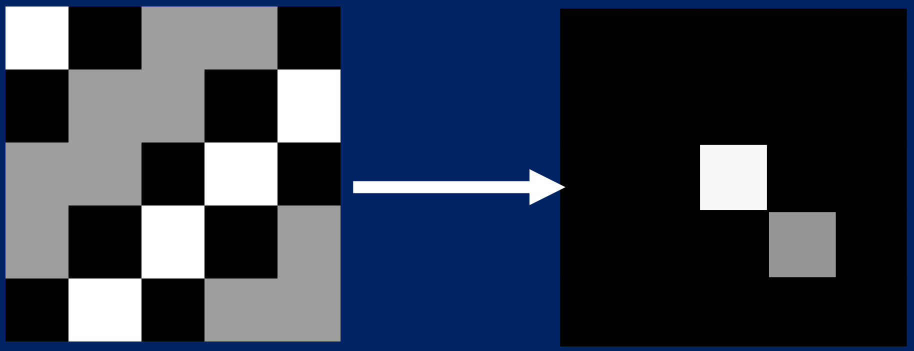
- Sparse Transformation
  - Fourier Transform
  - Wavelet (JPEG2000)
  - Discrete Cosine Transform (JPEG)
  - …etc

- Sparse Data



(13,64) (18,32)

- Transformed Sparse Data

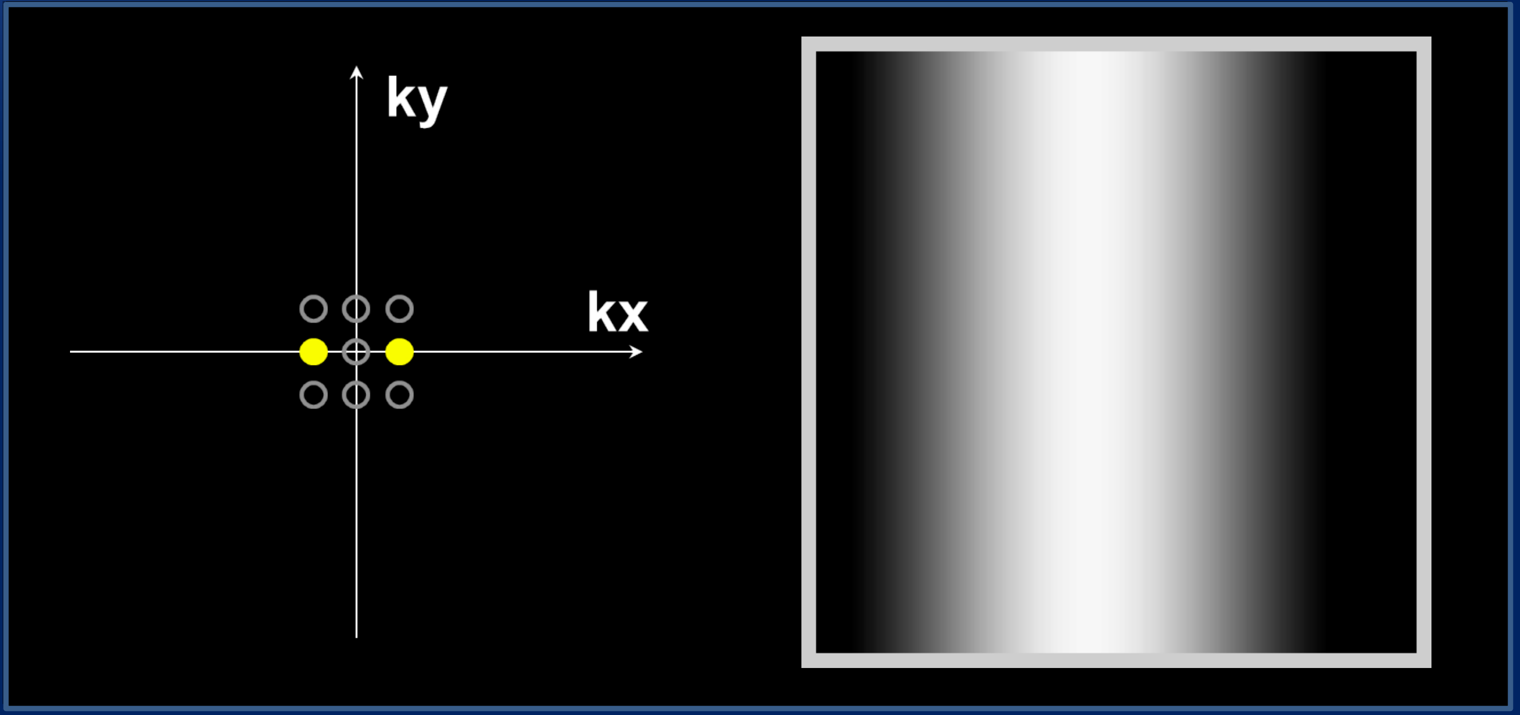


Compressive Sampling

- Sparse Representation
- Incoherent sampling & representation space
- Restricted Isometry Property

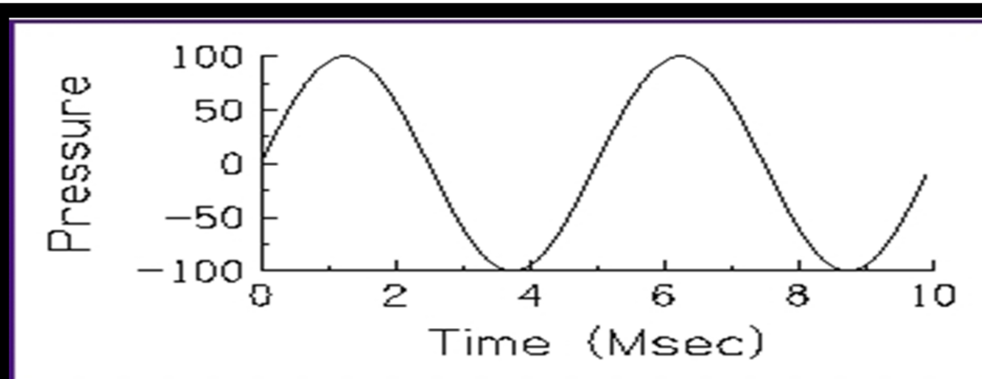
Incoherence

- Different behavior of the coefficients between the sensing basis (for k-space) and the representation bases (for ρ space)

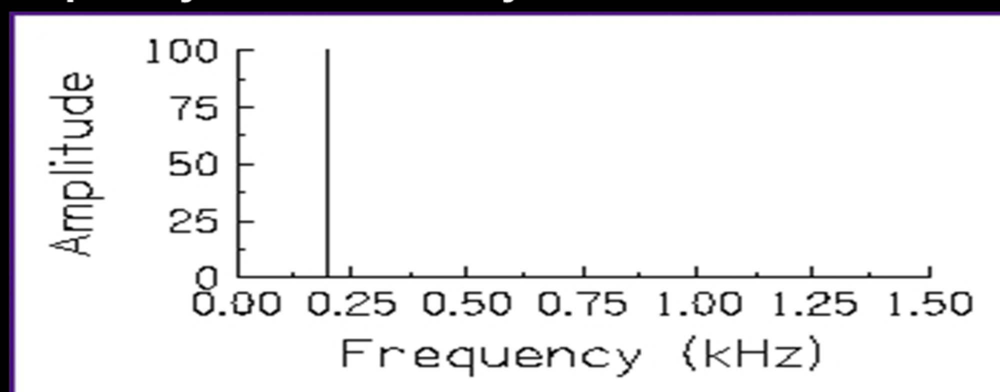


Incoherence

Time V.S. Frequency

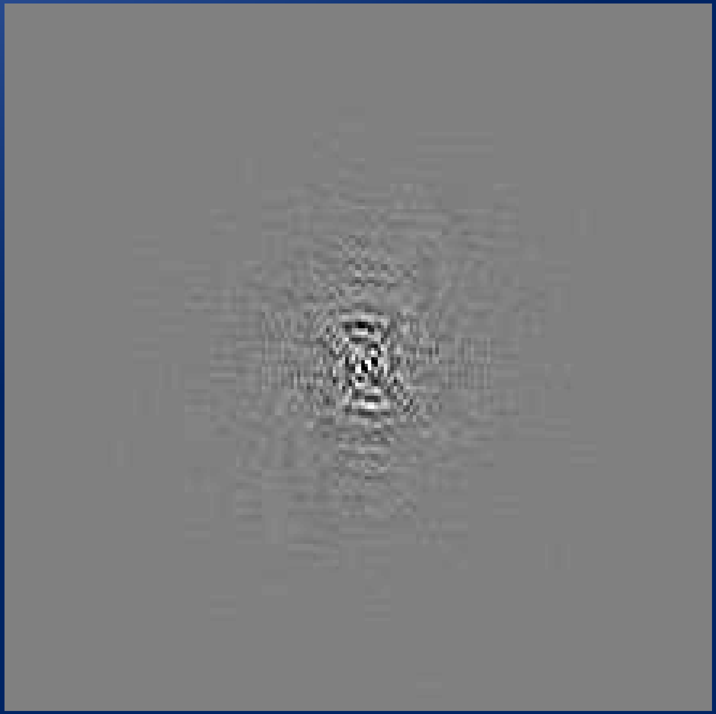


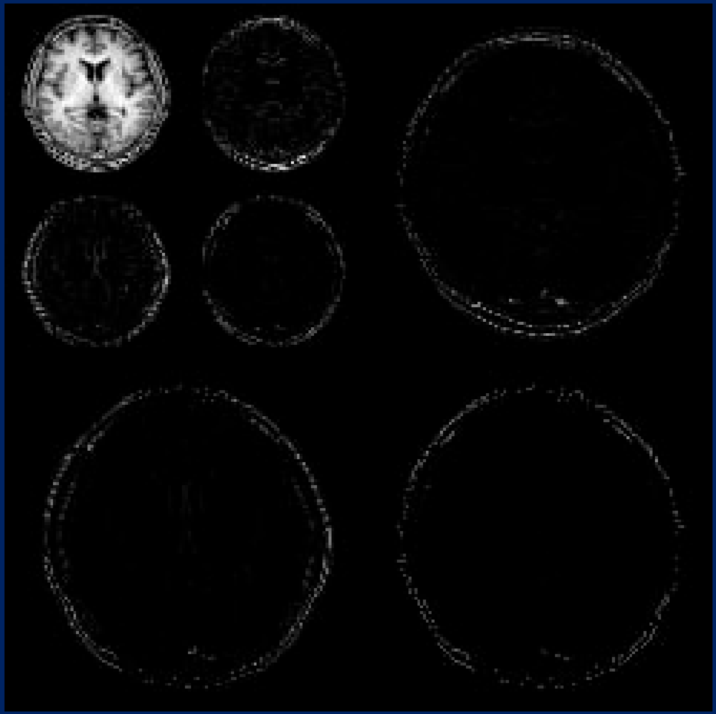
- **Amplitude = 100**
- **Frequency = number of cycles in one second = 200 Hz**



Incoherence

- k-space V.S Wavelet space





Compressive Sampling

- Sparse Representation
- Incoherent sampling & representation space
- Restricted Isometry Property

Restricted Isometry Property

- The encoding matrix of Compressive Sampling must satisfy Uniform Uncertainty Principle.

- Uniform Uncertainty Principle (UUP)
  - aka Restricted Isometry Property

$$1-\varepsilon \leq \frac{\left\|\Phi\Psi\alpha\right\|_2}{\left\|\Psi\alpha\right\|_2} = \frac{\left\|\Phi\rho\right\|_2}{\left\|\rho\right\|_2} \leq 1+\varepsilon$$

$$\varepsilon \geq 0$$

$$\alpha : K - sparse\ data$$

Restricted Isometry Property

$$1-\varepsilon \le \frac{\left\|\Phi\Psi\alpha\right\|_2}{\left\|\Psi\alpha\right\|_2} = \frac{\left\|\Phi\rho\right\|_2}{\left\|\rho\right\|_2} \le 1+\varepsilon$$

- K-sparse data are mostly distinguishable in Φ

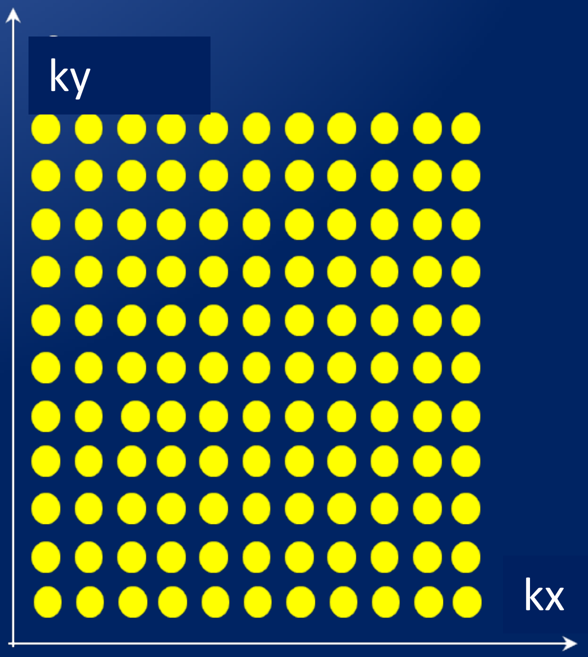
$$\rho_1 \ne \rho_2; \quad \Phi\rho_1 \ne \Phi\rho_2$$

- And the undersampled data in representation space may look similar to the original data

$$\Phi^{-1}\Phi\rho \approx \rho$$

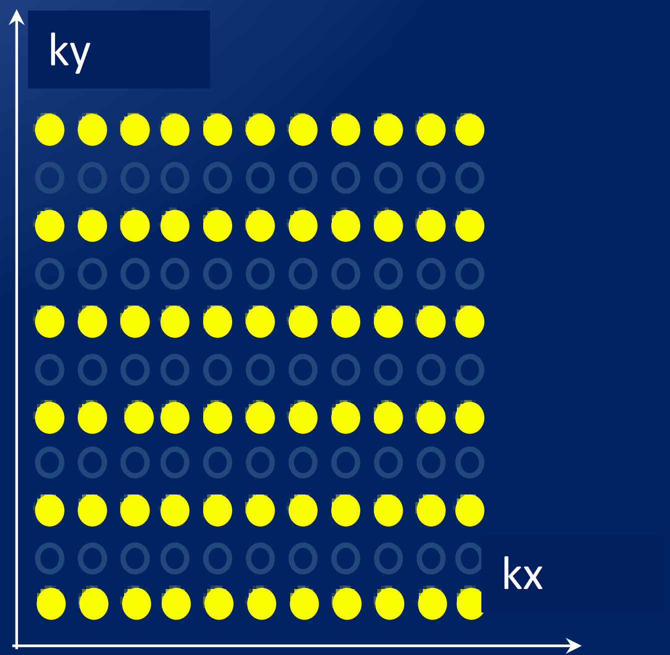
Restricted Isometry Property

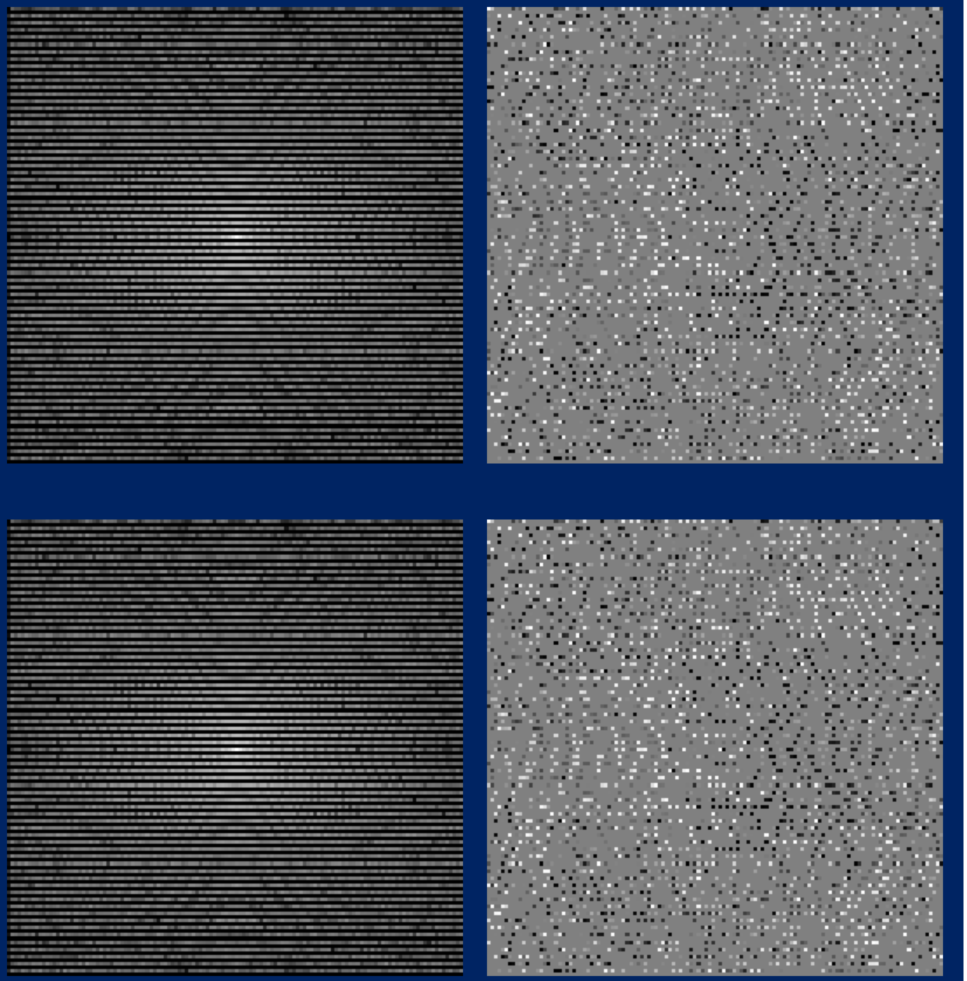
$$\rho_1 \neq \rho_2$$



NO Restricted Isometry Property

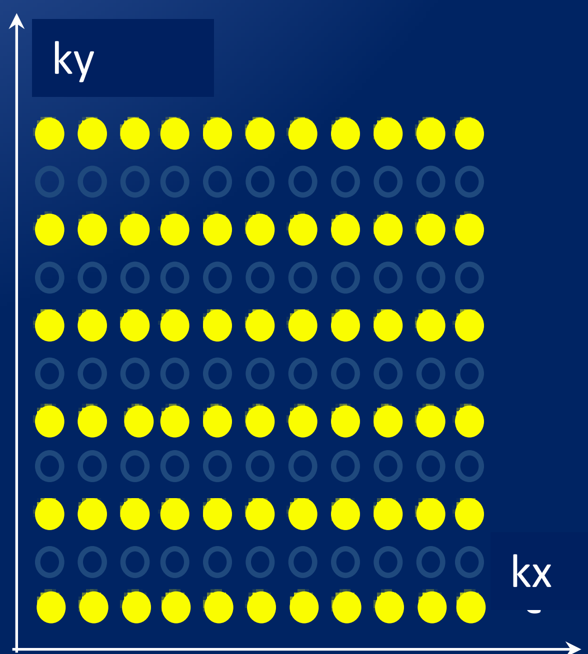
$$\Phi_s \rho_1 = \Phi_s \rho_2$$

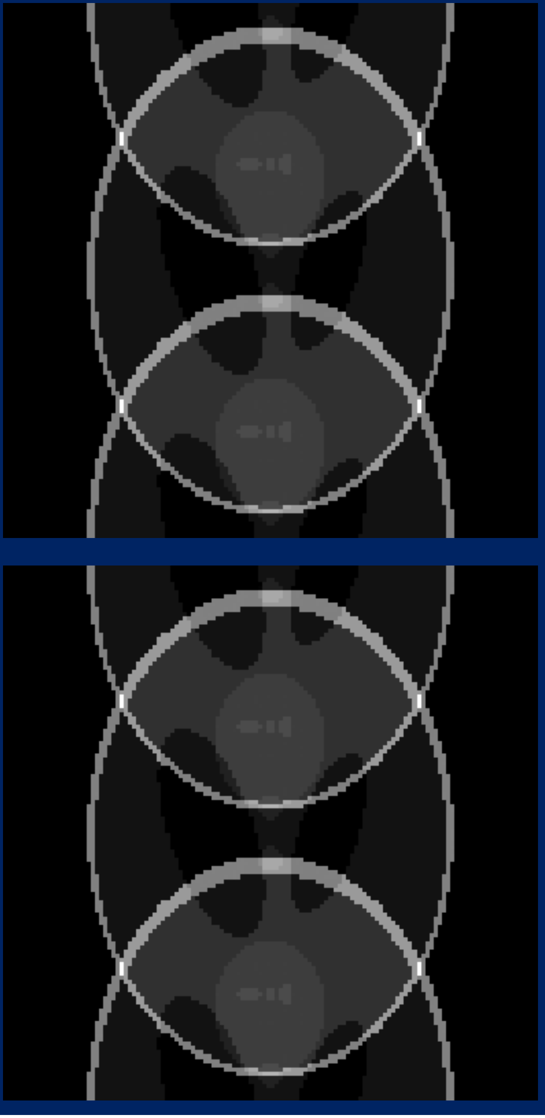




NO Restricted Isometry Property

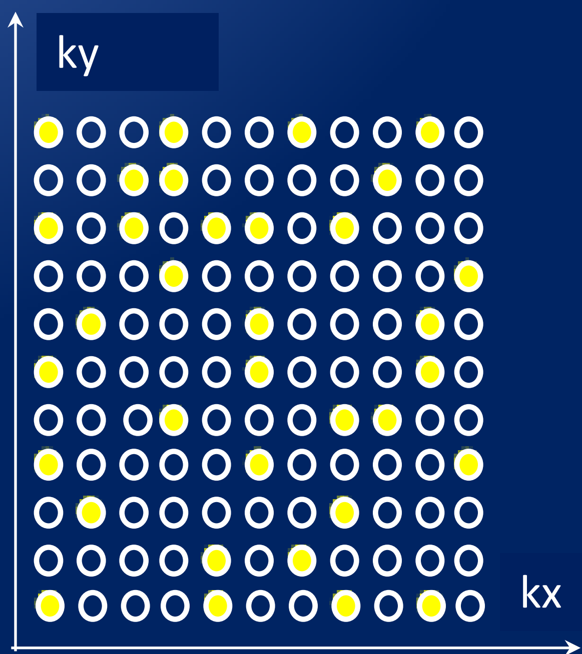
$$\Phi_s^{-1}\Phi_s\rho_1 = \Phi_s^{-1}\Phi_s\rho_2$$

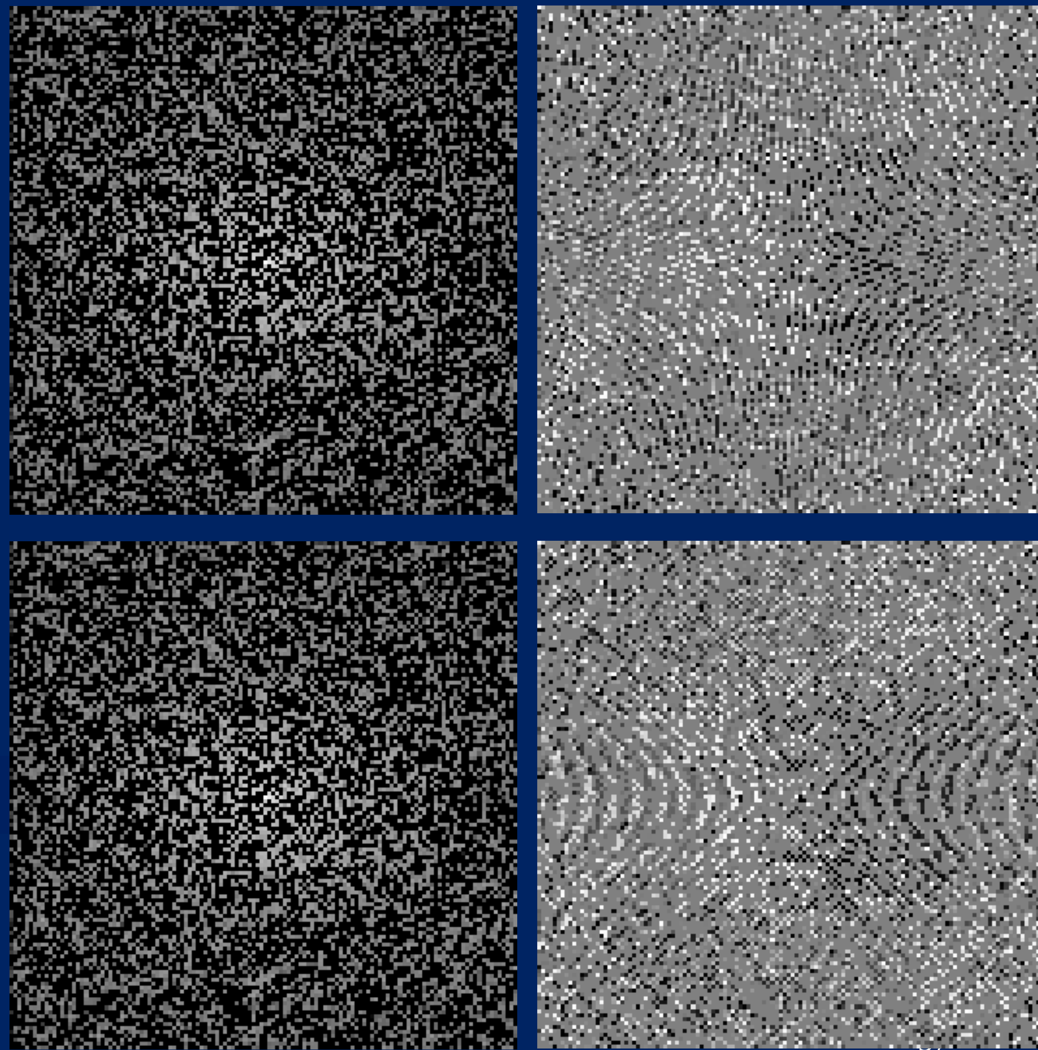




Restricted Isometry Property

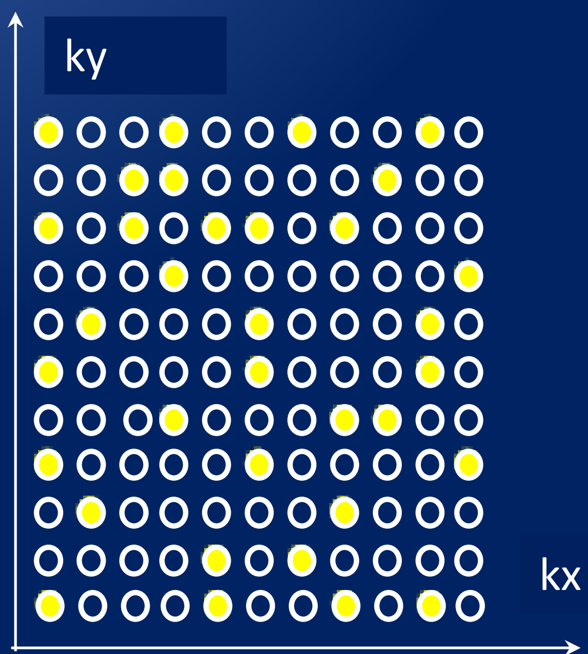
$$\Phi_r \rho_1 \neq \Phi_r \rho_2$$

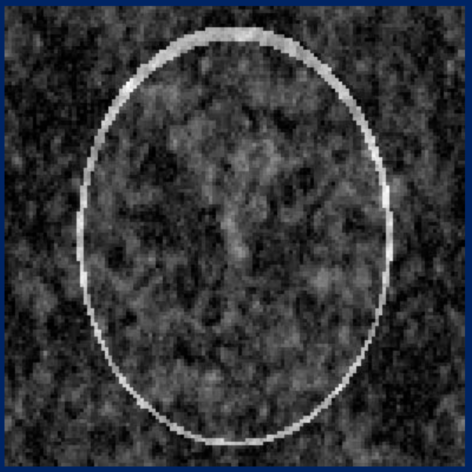


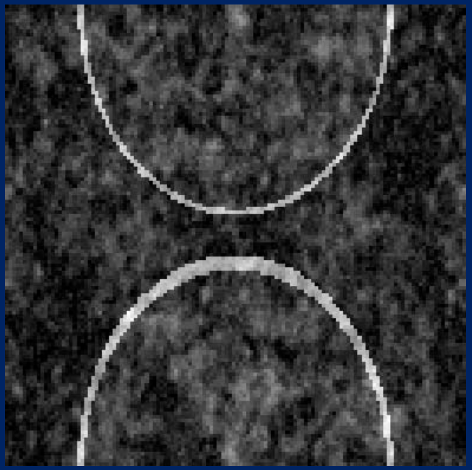


Restricted Isometry Property

$$\Phi_r^{-1}\Phi_r\rho_1 \neq \Phi_r^{-1}\Phi_r\rho_2$$

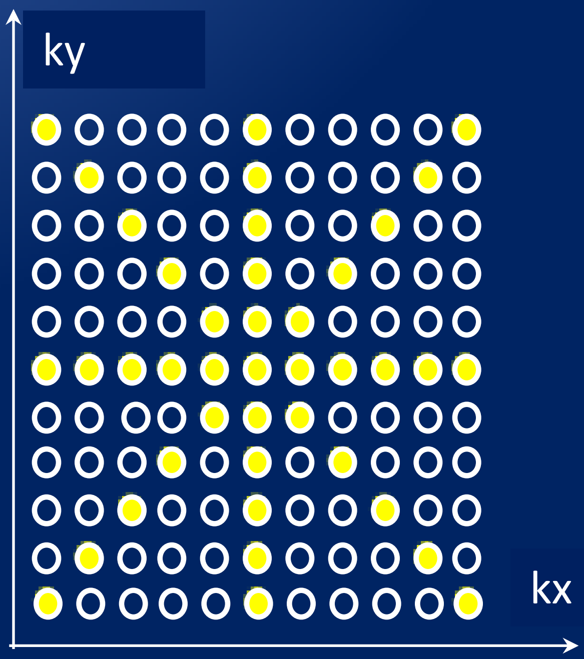


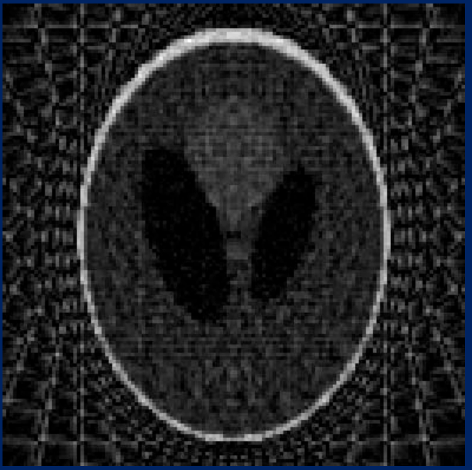


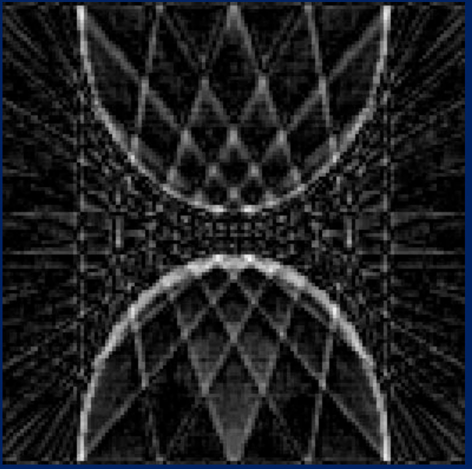


Restricted Isometry Property

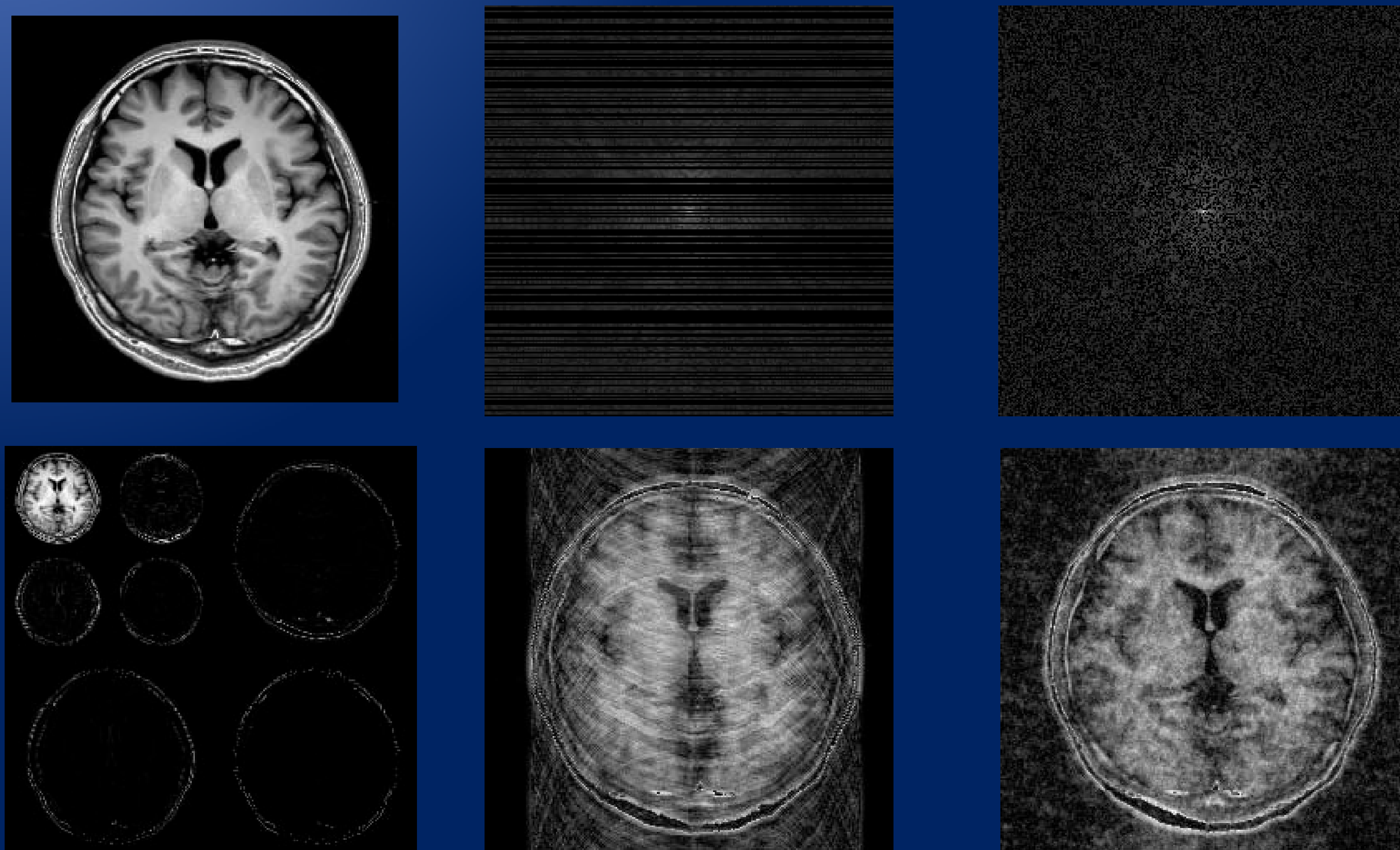
$$\Phi_r^{-1}\Phi_r\rho_1 \neq \Phi_r^{-1}\Phi_r\rho_2$$



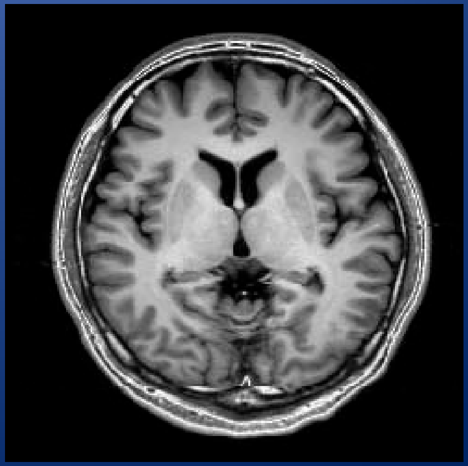
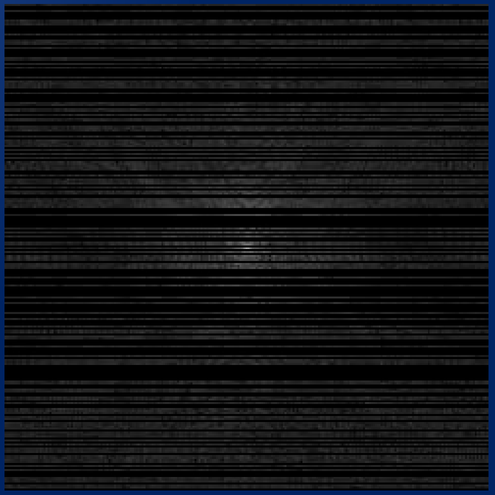
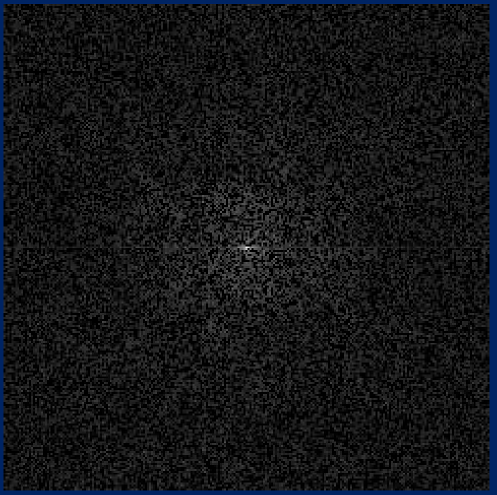
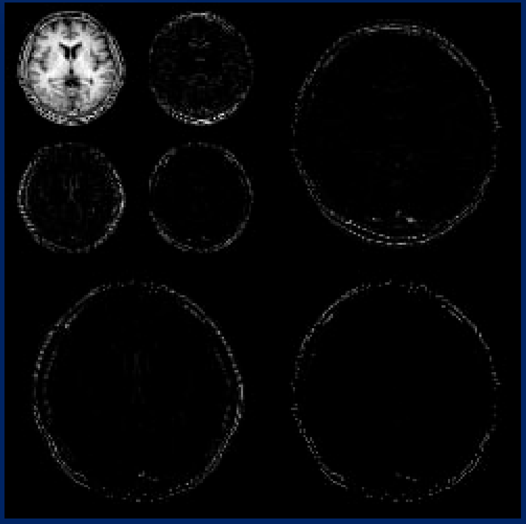
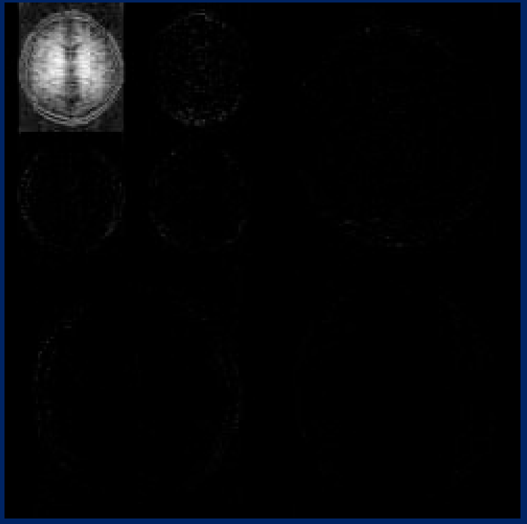
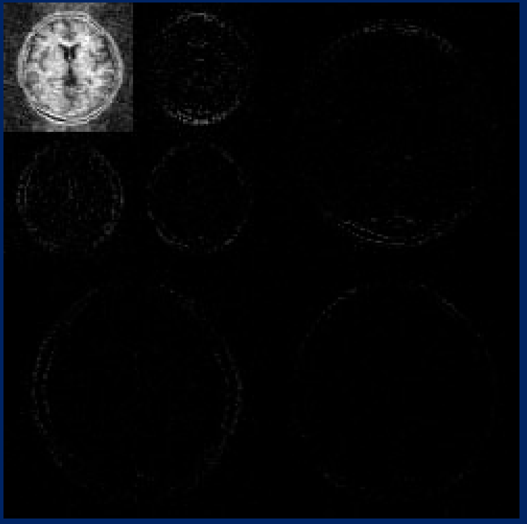




Toward Compressed Sensing



Toward Compressed Sensing

Compressed Sensing

- Regularly Undersampling
  - Regularly distributed aliasing artifact
  - Difficult to distinguish signal and artifacts
- Irregularly Undersampling
  - Irregular distributed artifacts (Noise Like)
  - Noise suppression algorithm may help
  - How about finding a sparsest solution

Compressed Sensing

- Sparse Representation
- Compressive Sampling
- Signal Recovery

Finding a Solution

- Image Encoding

$$\mathbf{k} \quad = \mathbf{\Phi\Psi}\boldsymbol{\alpha} = \mathbf{E}\boldsymbol{\alpha}$$

- Image Decoding

$$\boldsymbol{\alpha}' = (E^T E)^{-1} E^T \mathbf{k}$$

or

$$\boldsymbol{\alpha} = min_{\alpha'} |\mathbf{k} - \mathbf{E}\boldsymbol{\alpha}'|_2^2$$

k : Data
Φ : Encoding
Ψ: Sparsification
$\boldsymbol{\alpha}$: Sparse Signal

Finding a CS Solution

- Encoding

$$\mathbf{k} \quad = \boldsymbol{\Phi\Psi}\boldsymbol{\alpha} = \mathbf{E}\boldsymbol{\alpha}$$

k : Data
Φ : Encoding
Ψ: Sparsification
$\boldsymbol{\alpha}$: Sparse Signal

- Decoding

Find a solution $\boldsymbol{\alpha}'$ such that

$\Psi\boldsymbol{\alpha}'$ is the sparsest

$$\& \qquad k - E\boldsymbol{\alpha}' = 0$$

An example of CS solution

- A sparse signal:

$$x = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \quad ;$$

represented in the bases $[1,0,0], [0,1,0]$ *and* $[0,0,1]$

- The signal was encoded by a new random bases

$$\Phi = \begin{bmatrix} 0.2 & 0.49 & 0.46 \\ 0 & 0 & 0 \\ 0.61 & 0.77 & 0.83 \end{bmatrix} \qquad y = \Phi x = \begin{bmatrix} 0.98 \\ 0 \\ 1.54 \end{bmatrix}$$

An example of CS solution

- Now forget the x.
  Let's get the x back from the encoded signal y and the encoding matrix Φ.

$$\Phi = \begin{bmatrix} 0.2 & 0.49 & 0.46 \\ 0 & 0 & 0 \\ 0.61 & 0.77 & 0.83 \end{bmatrix} \qquad y = \Phi x = \begin{bmatrix} 0.98 \\ 0 \\ 1.54 \end{bmatrix}$$

An example of CS solution

- What we know about x
  - 3 elements $[x_1, x_2, x_3]$
  - a solution of the $y = \Phi x$
  - And it's sparse

$$\begin{bmatrix} 0.98 \\ 0 \\ 1.54 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.49 & 0.46 \\ 0 & 0 & 0 \\ 0.61 & 0.77 & 0.83 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

An example of CS solution

- Parametric form of the general solutions of x

$$x' = \Phi^{-1} y + t\,\mathbf{ker}(\Phi)$$

$$= \begin{bmatrix} -0.3262 \\ 1.2879 \\ 0.9004 \end{bmatrix} - t \begin{bmatrix} -0.2734 \\ -0.5967 \\ 0.7545 \end{bmatrix}$$

The answer shall be the sparsest

- If $x_1 = 0$, x = [0, 2, 0]
- If $x_2 = 0$, x = [0.91, 0, 2.5289]

$$x' = \Phi^{-1} y + t\,\mathbf{ker}(\Phi)$$

$$= \begin{bmatrix} -0.3262 \\ 1.2879 \\ 0.9004 \end{bmatrix} - t \begin{bmatrix} -0.2734 \\ -0.5967 \\ 0.7545 \end{bmatrix}$$

Searching in solution space

- Exactly recover the precisely sparse data
- The only method compatible to find the solution subject to min L0-norm
- Time consuming
- Not suitable for slightly larger scale system

Finding a Sparsest Solution

- Image Encoding $\mathbf{k} \quad = \boldsymbol{\Phi}\boldsymbol{\rho} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} = \mathbf{E}\boldsymbol{\alpha}$

- *L0 or L1*-norm Regularized Image Decoding

$$minimize \quad |\boldsymbol{\alpha}'|_p \text{ subject to} |\mathbf{k} - \mathbf{E}\alpha'|_2 < \epsilon$$

or

$$\rho' = min_{\rho'} |\mathbf{k} - \mathbf{E}\alpha'|_2^2 + \lambda |\boldsymbol{\alpha}'|_p^p$$

Finding a Sparsest Solution

- Image Encoding $\mathbf{k} = \boldsymbol{\Phi}\boldsymbol{\rho} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} = \mathbf{E}\boldsymbol{\alpha}$
- *L0 or L1*-norm Regularized Image Decoding

$$minimize \quad \left|\boldsymbol{\Psi}^{-1}\boldsymbol{\rho}'\right|_p \text{ subject to} |\mathbf{k} - \boldsymbol{\Phi}\boldsymbol{\rho}'|_2 < \epsilon$$

or

$$\rho' = min_{\rho'} |\mathbf{k} - \boldsymbol{\Phi}\boldsymbol{\rho}'|_2^2 + \lambda\left|\boldsymbol{\Phi}^{-1}\boldsymbol{\rho}'\right|_p^p$$

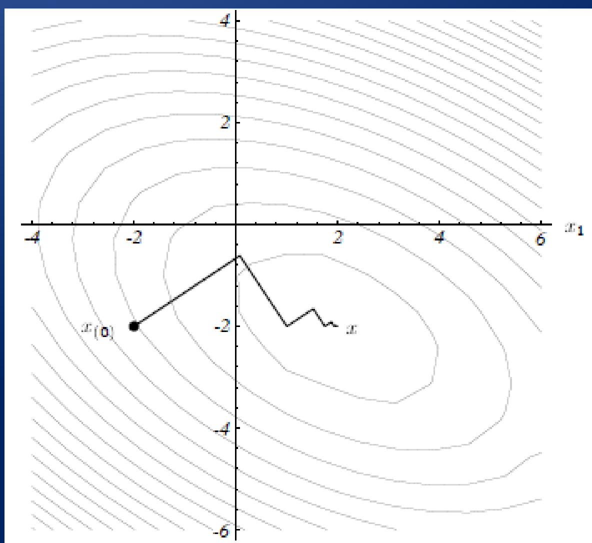
Algorithms to explore CS solution

- Sparsest searching (impractical)
- non-linear Conjugate Gradient
  - CG: Newton’s Method in matrix form
  - line search
- Basis pursuit (A greedy method)
  - Orthogonal Matching Pursuit
- LASSO: least absolute shrinkage and selection operator
- ……… (FOCUSS)

non-linear Conjugate Gradient

$$\underset{m}{\text{argmin}} \quad \|\mathcal{F}_u m - y\|_2{}^2 + \lambda \|\Psi m\|_1,$$

$$\nabla f(m) = 2F_u^*(F_u m - y) + \lambda \nabla ||\Psi m||_1$$



% Initialization

$k = 0;\ m = 0;\ g_0 = \nabla f(m_0);\ \Delta m_0 = -g_0$

% Iterations

while $(||g_k||_2 < \text{TolGrad and } k > \text{maxIter})$ {

% Backtracking line-search

$t = 1;$ *while* $(f(m_k + t\Delta m_k) \; > \; f(m_k) + \alpha t \cdot Real(g_k^* \Delta m_k))$

$\{t = \beta t\}$

$m_{k+1} = m_k + t\Delta m_k$

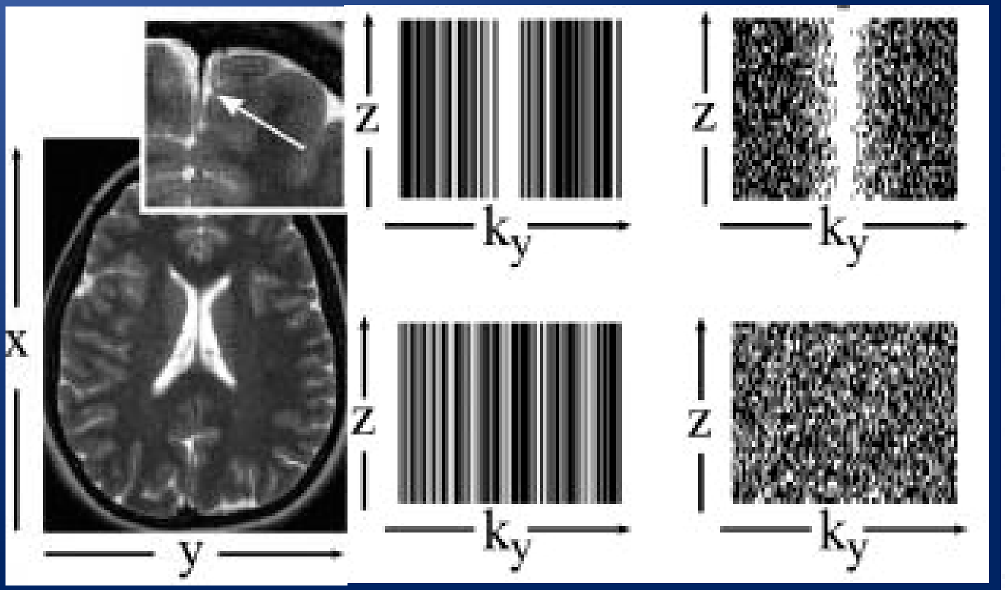
$g_{k+1} = \nabla f(m_{k+1})$

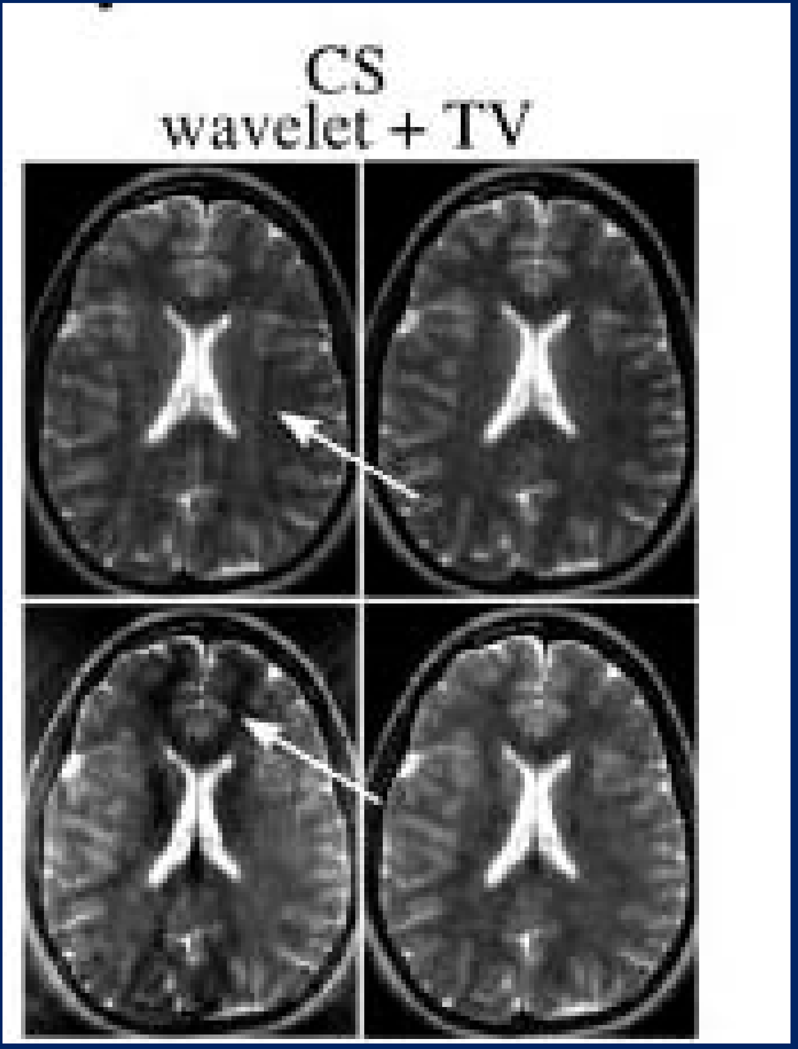
$\gamma = \frac{||g_{k+1}||_2^2}{||g_k||_2^2}$

$\Delta m_{k+1} = -g_{k+1} + \gamma \Delta m_k$

$k = k + 1$ }

SPARSE MRI

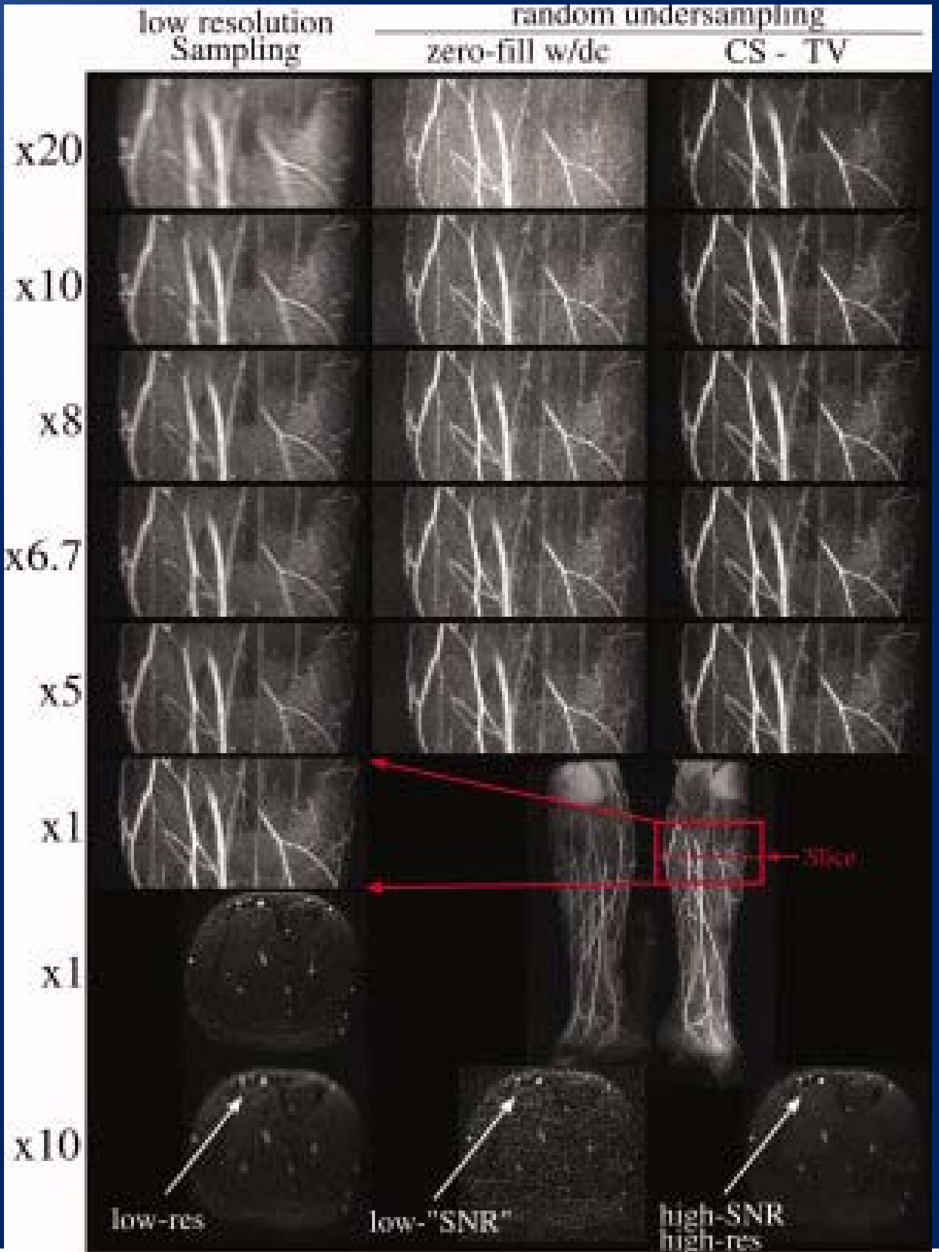




Michael Lustig et al.

Magn Reson Med, 58: 1182–1195 (2007)

SPARSE MRI



Michael Lustig et al.

Magn Reson Med, 58: 1182–1195 (2007)

SPARSE MRI

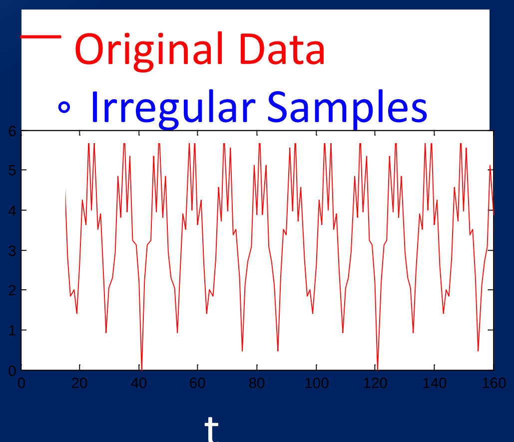
- The sparser, the better
  - Contrast Enhanced MRA has better reconstruction than conventional T1w and T2w images.
- Higher dimension has more degrees of freedom for sparse representation
  - Better reconstruction in 3D MRI than in 2D

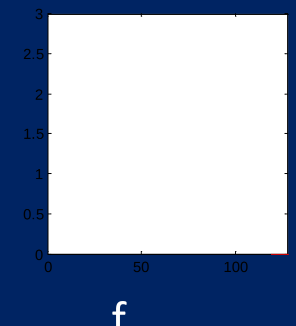
Algorithms to explore CS solution

- Sparsest searching (impractical)
- non-linear Conjugate Gradient
  - CG: Newton's Method in matrix form
  - line search
- Basis pursuit (A greedy method)
  - Orthogonal Matching Pursuit
- LASSO: least absolute shrinkage and selection operator
- ……… (FOCUSS)

Data Recovery: Greedy Method

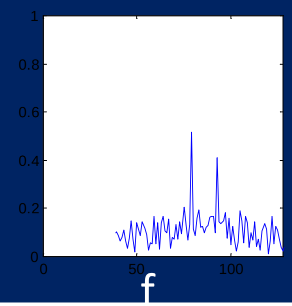
- Orthogonal Matching Pursuit
- Keep picking up the maximal signal in the sparse representation of each iteration
- Remove the component from the undersampled data

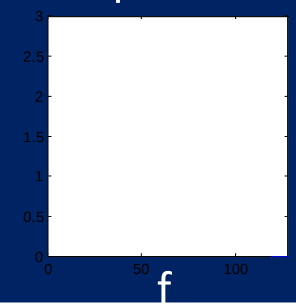




Original Sparse Information

Lease Square Reconstruction

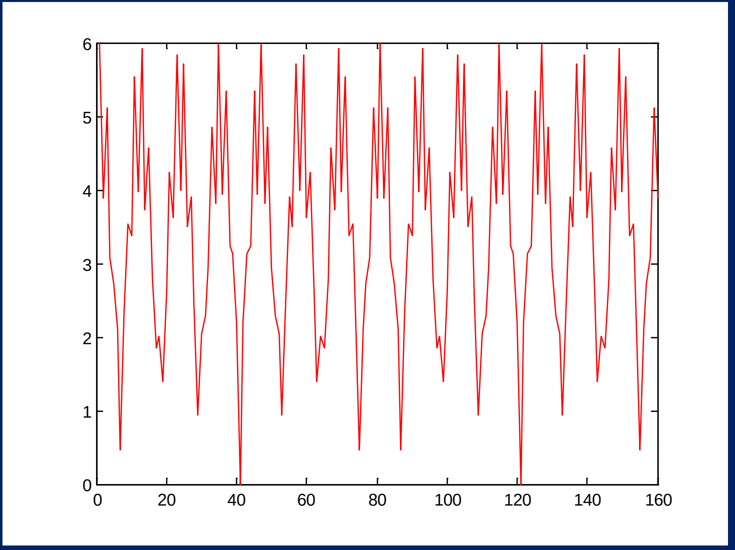




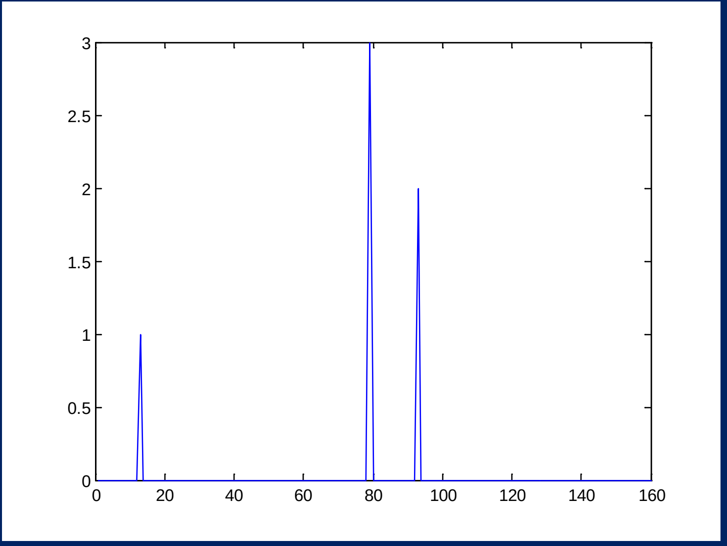
Reconstructed Information

Data Recovery: Greedy Method

- Original fully-sampled acquisition

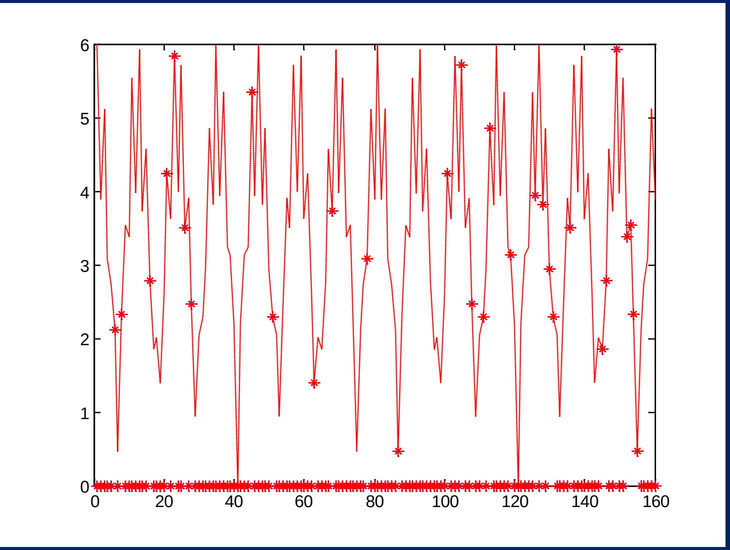


- Least square reconstruction

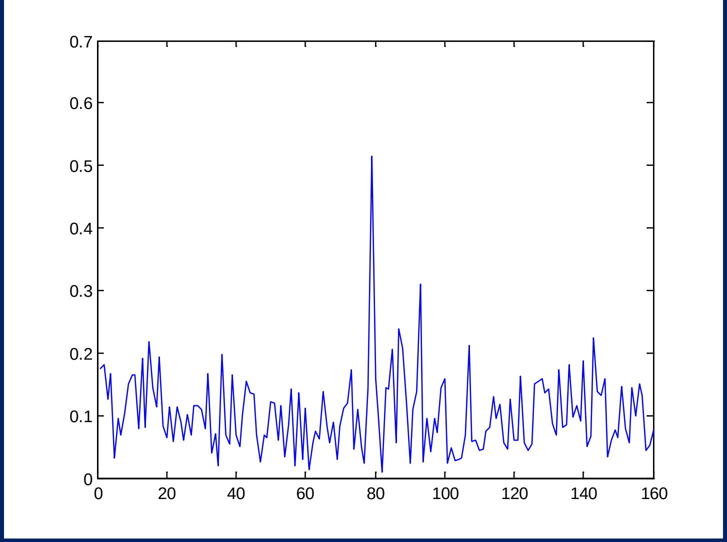


Data Recovery: Greedy Method

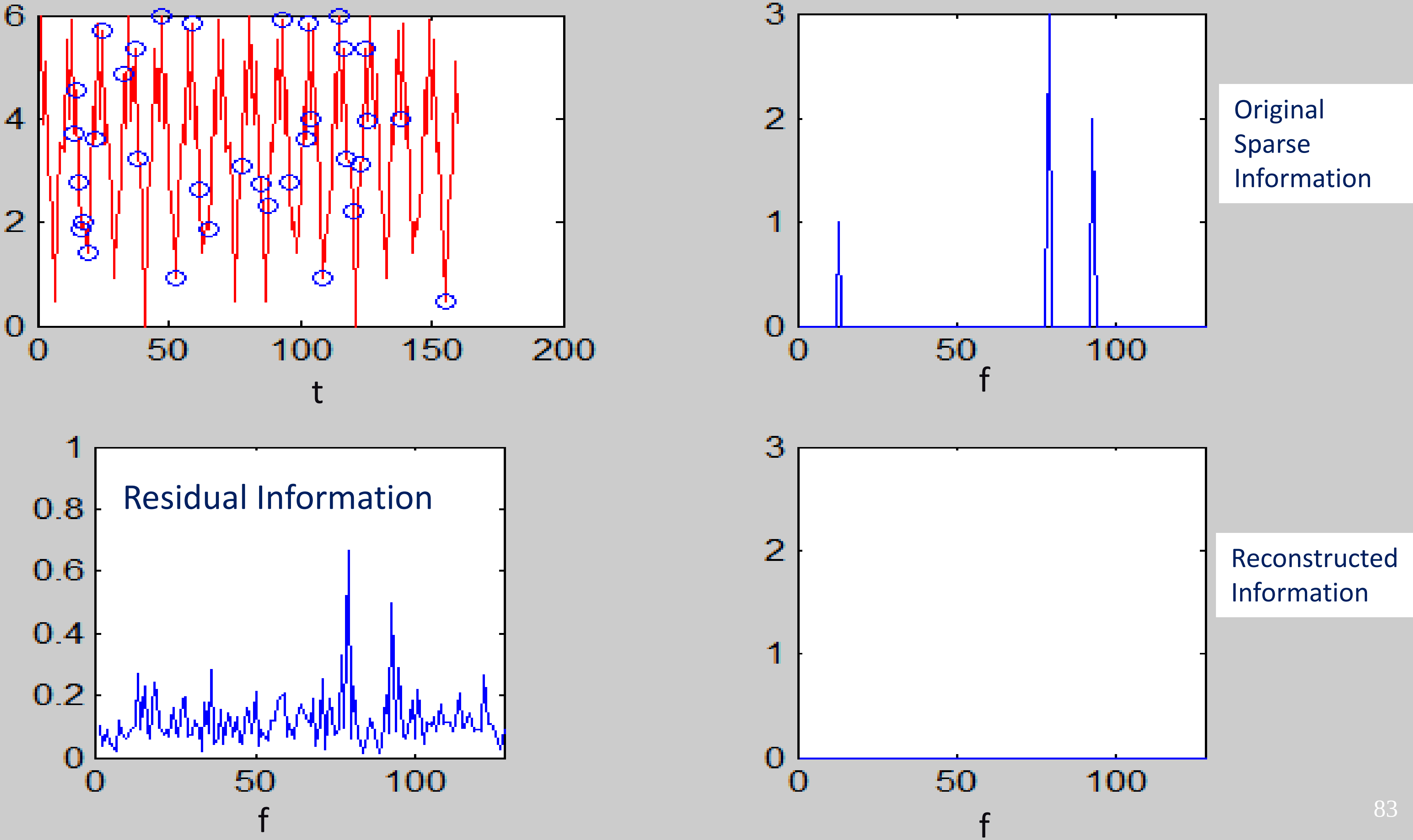
- Randomly undersampled acquisition



- Least square reconstruction

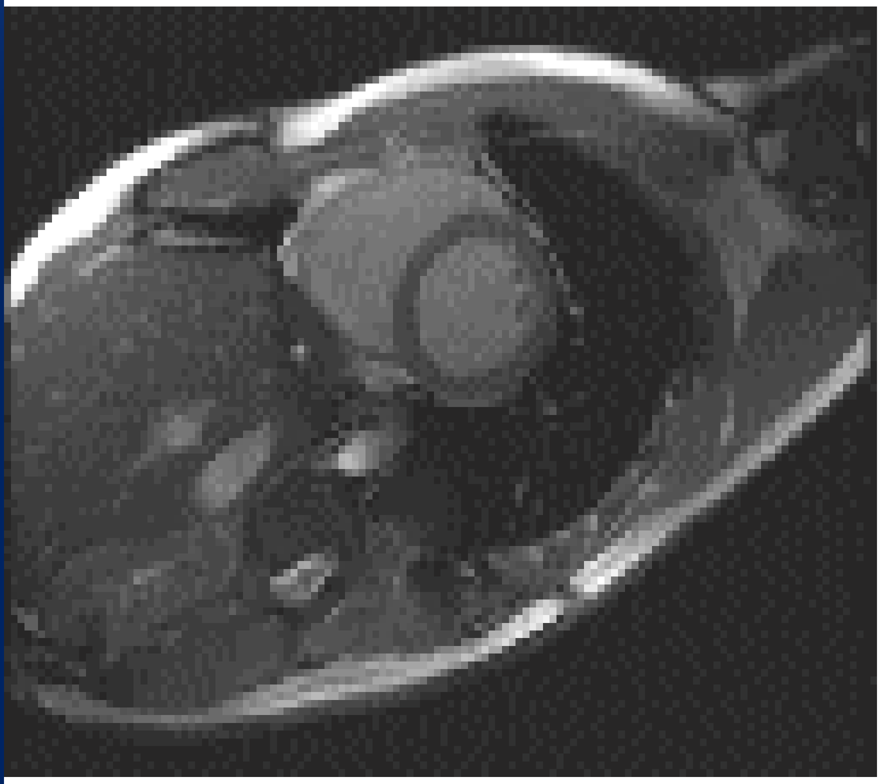


Data Recovery: Greedy Method

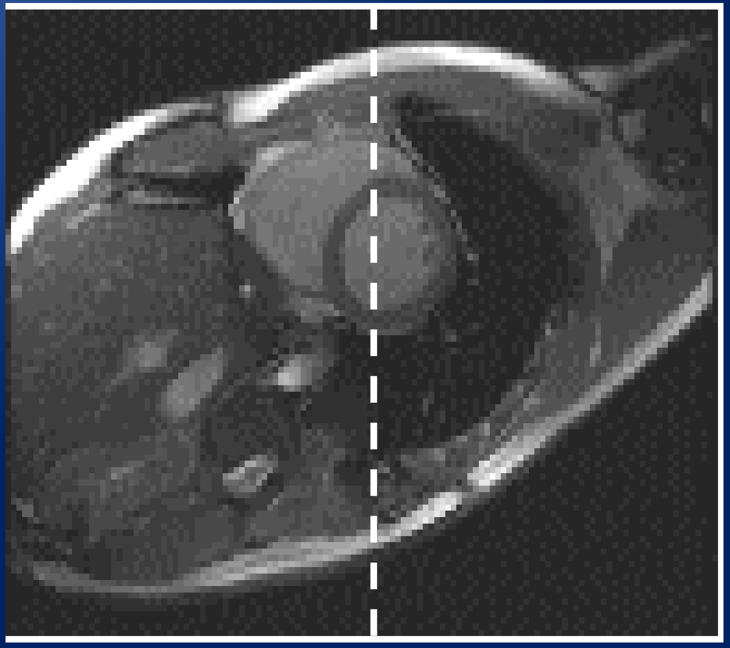


CS on Cardiac CINE imaging

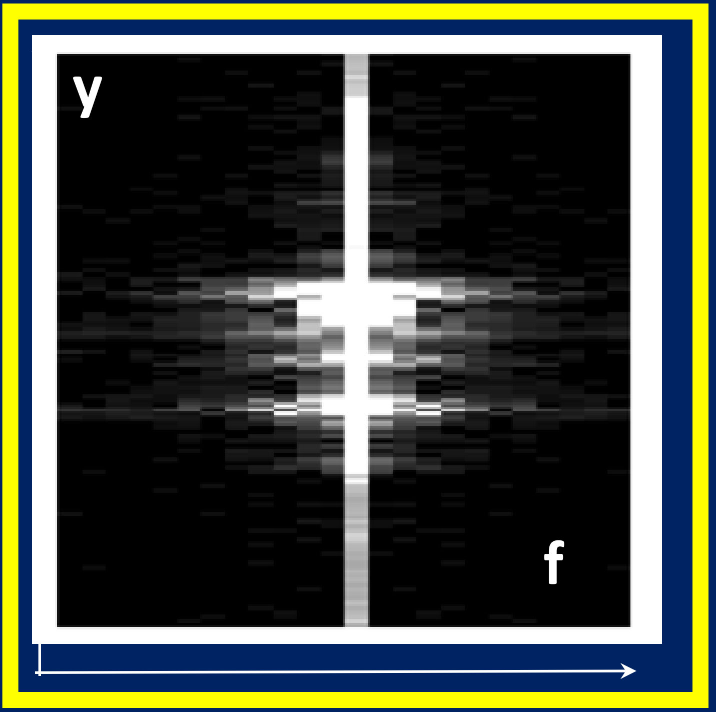
- Original FULLY sampled cardiac CINE



The Spectrum of Cardiac Motion

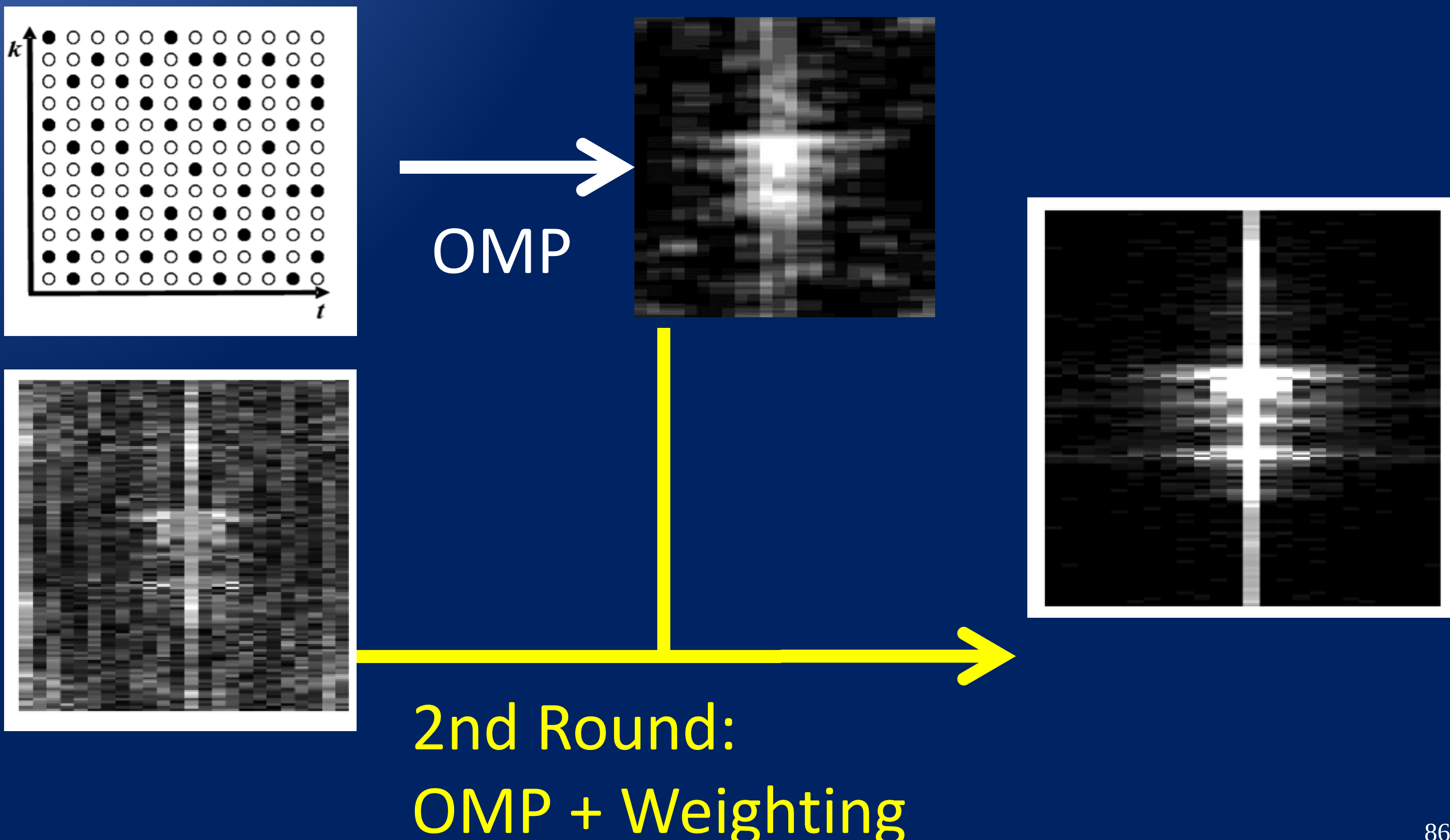


Cardiac CINE Image

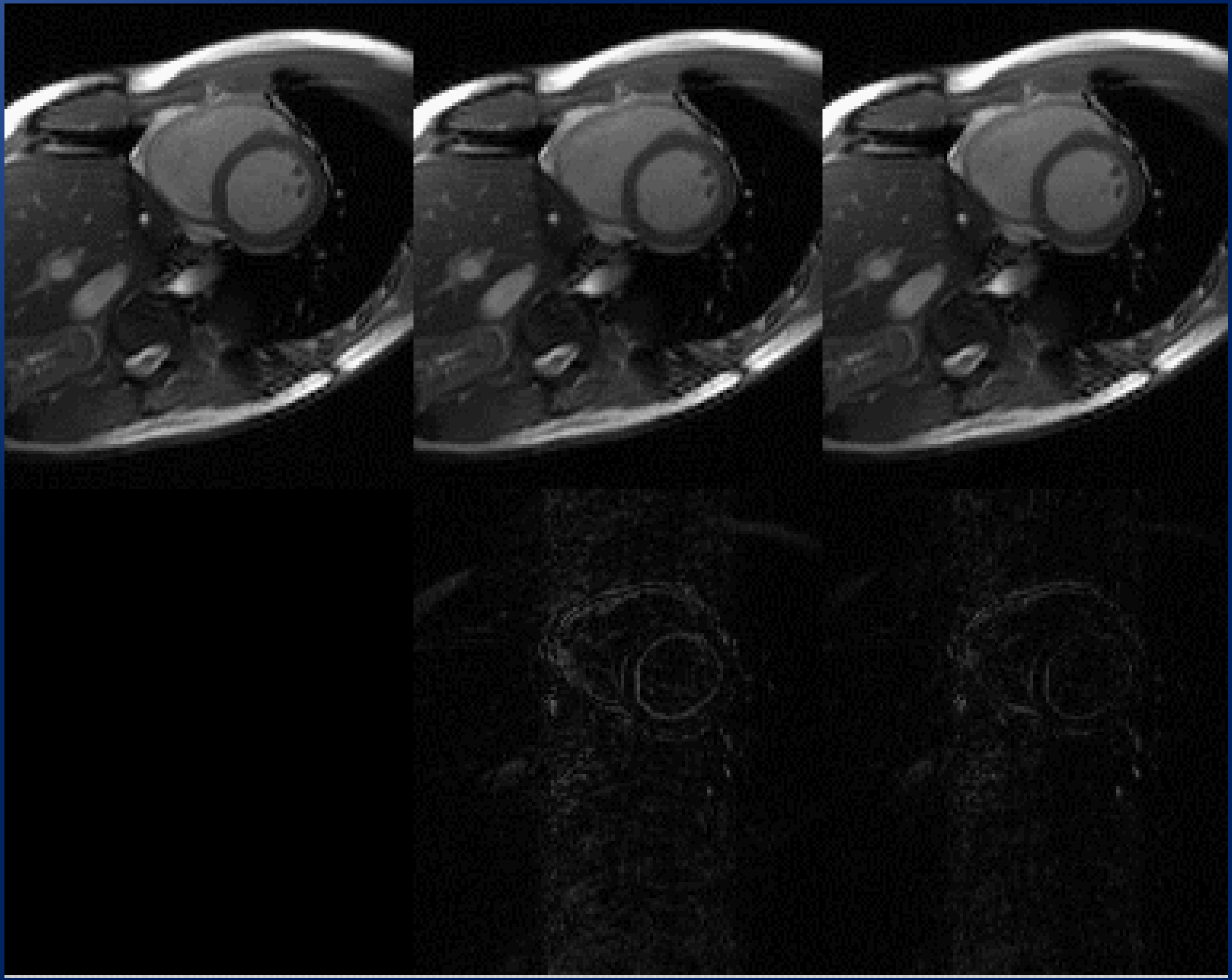


y-f spectrum

Reconstruction



w/ and w/o penalty (3x)



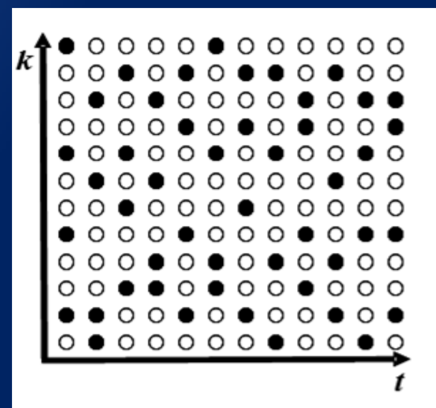
Reference Fully Sampled

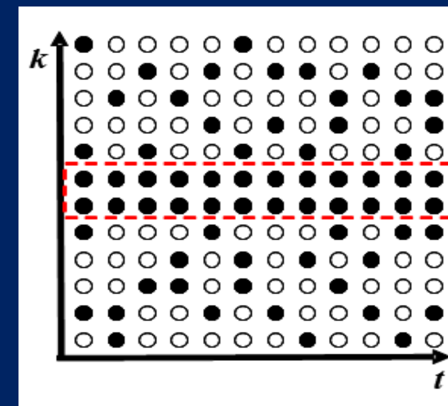
Without Penalty

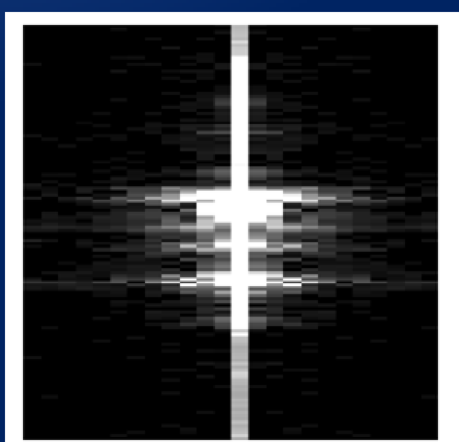
W/ Penalty from 1st recon

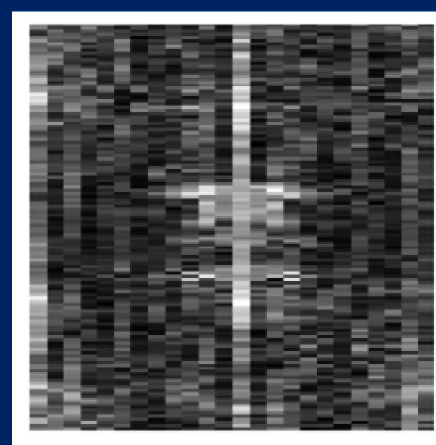
Fully Sampled V.S CS Sampled Spectrum of Cardiac Motion

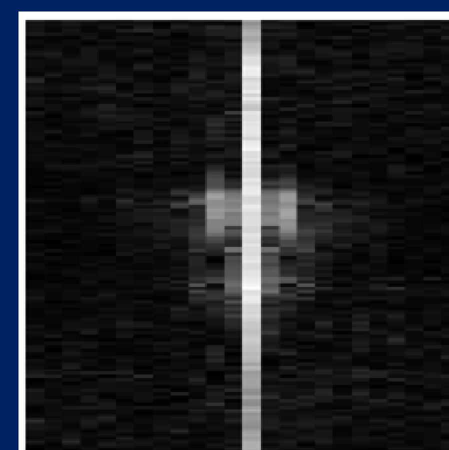
Fully Sampled Reference



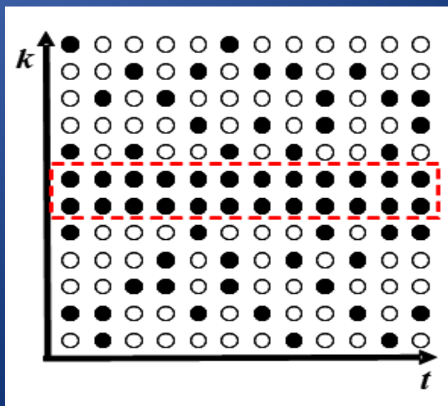




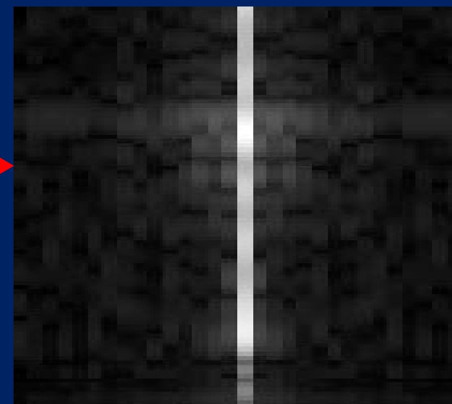


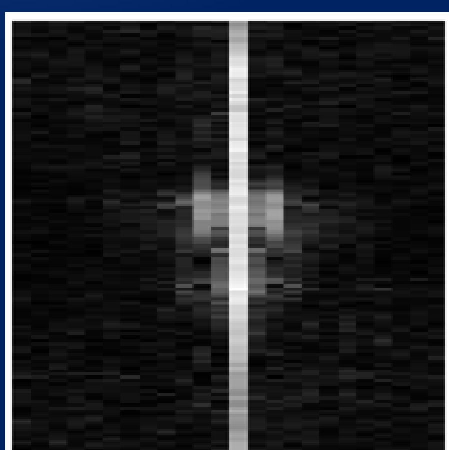


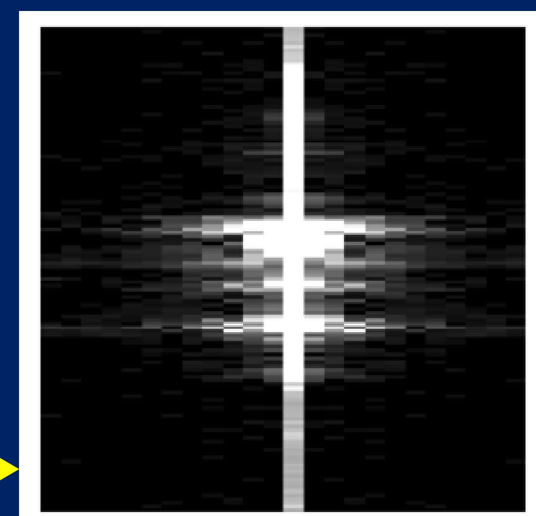
Reconstruction



FT

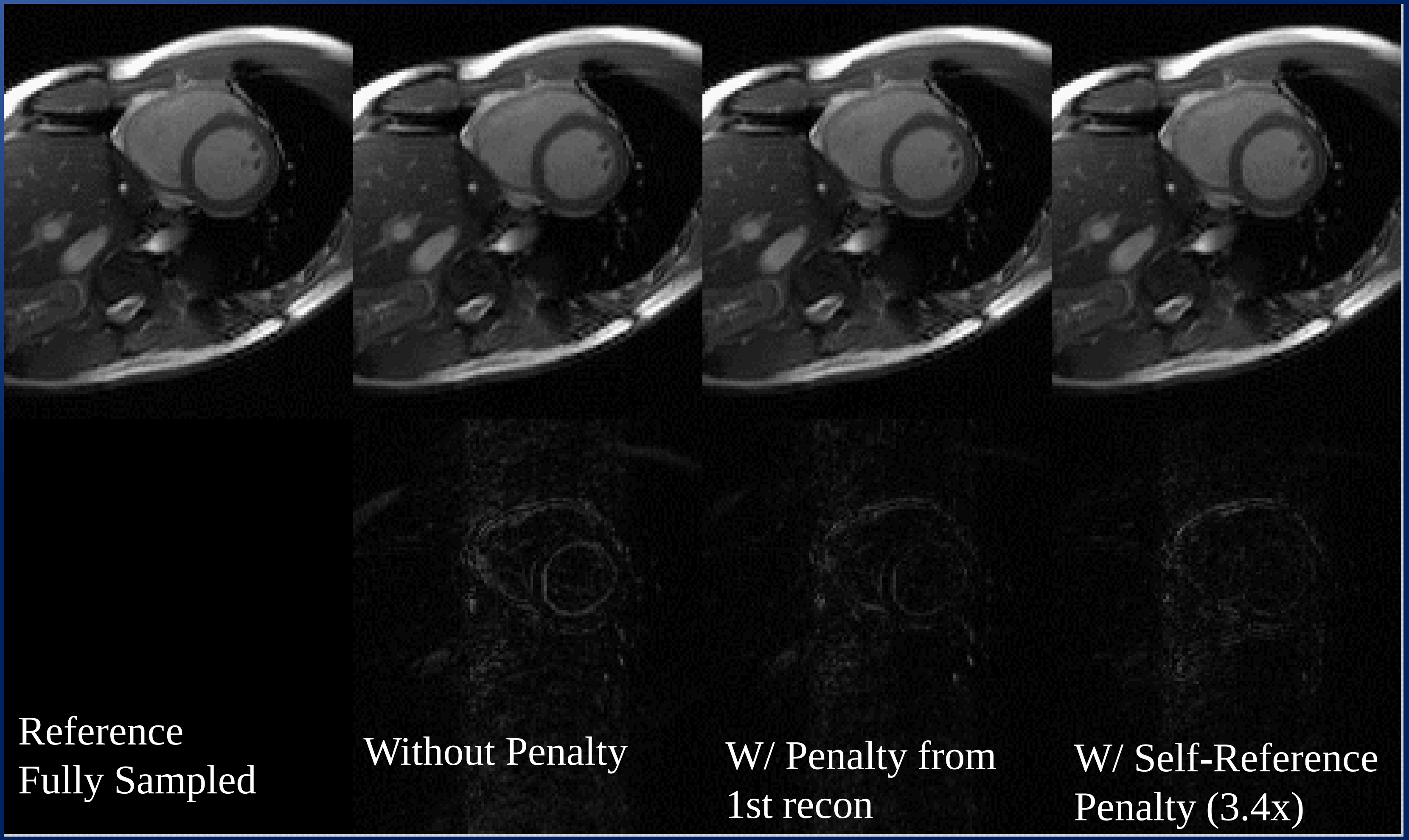




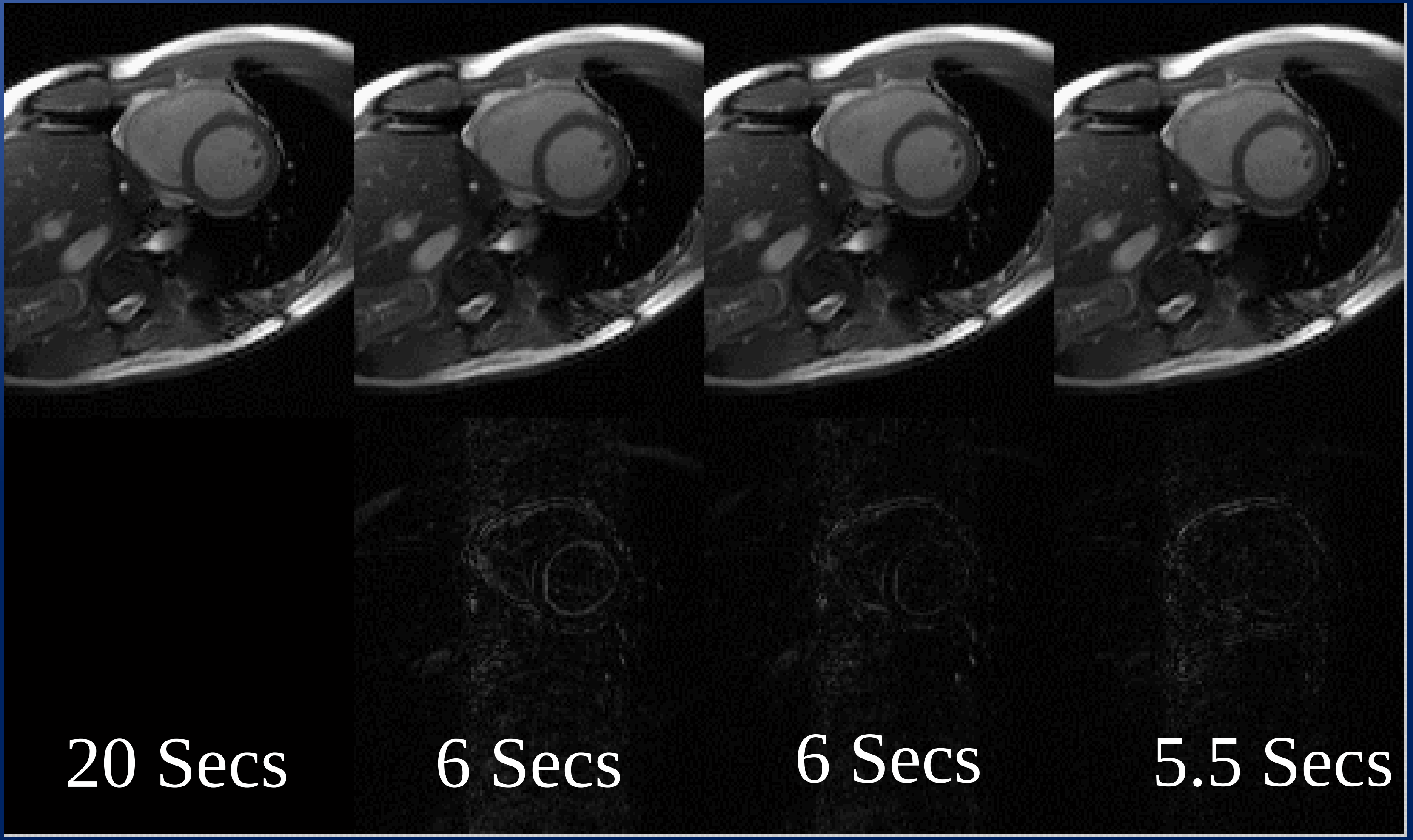


One Round:
OMP + Weighting

w/ and w/o penalty (3x)



w/ and w/o penalty (3x)



FMRI

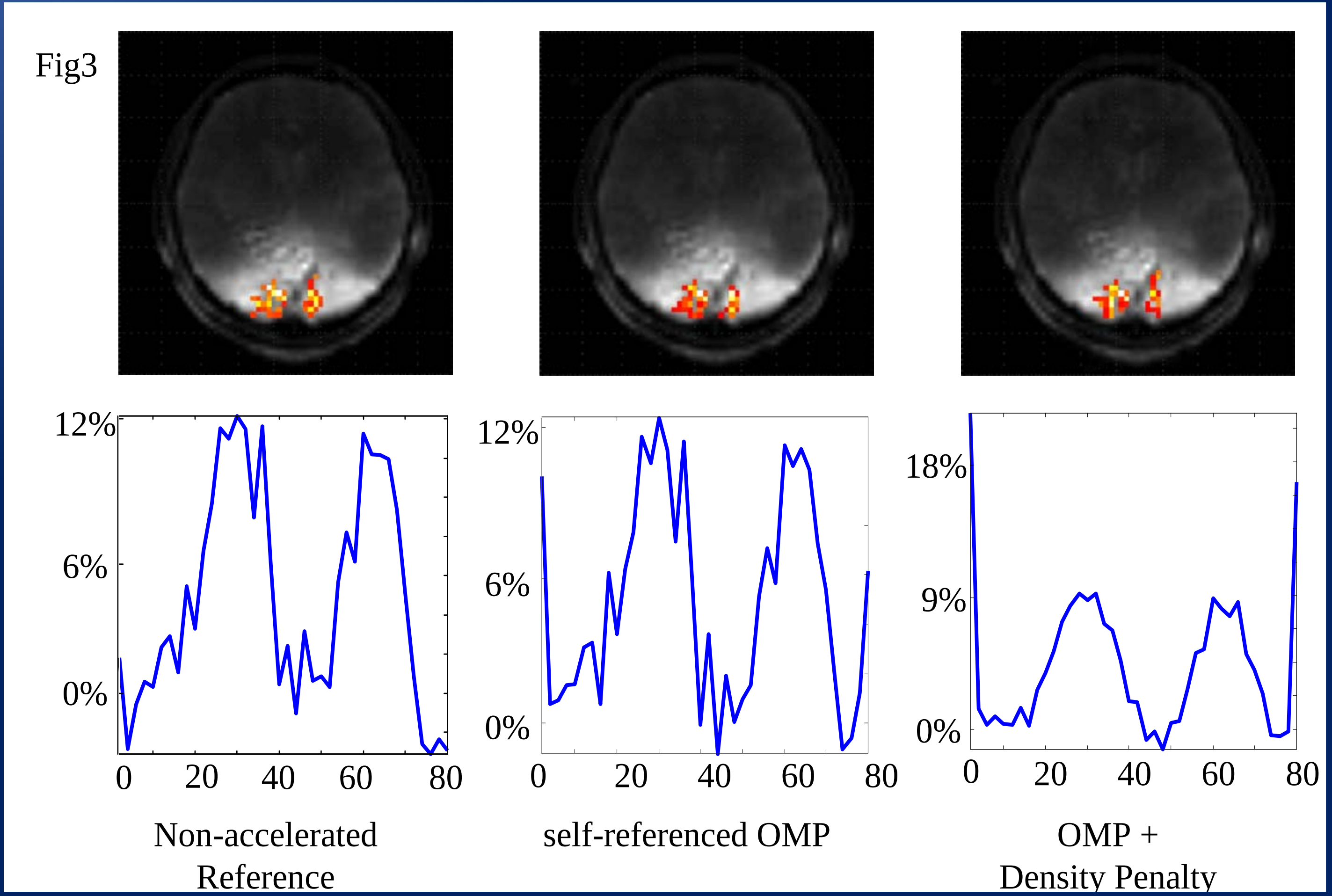


Fig3
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Non-accelerated
Reference
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80
self-referenced OMP
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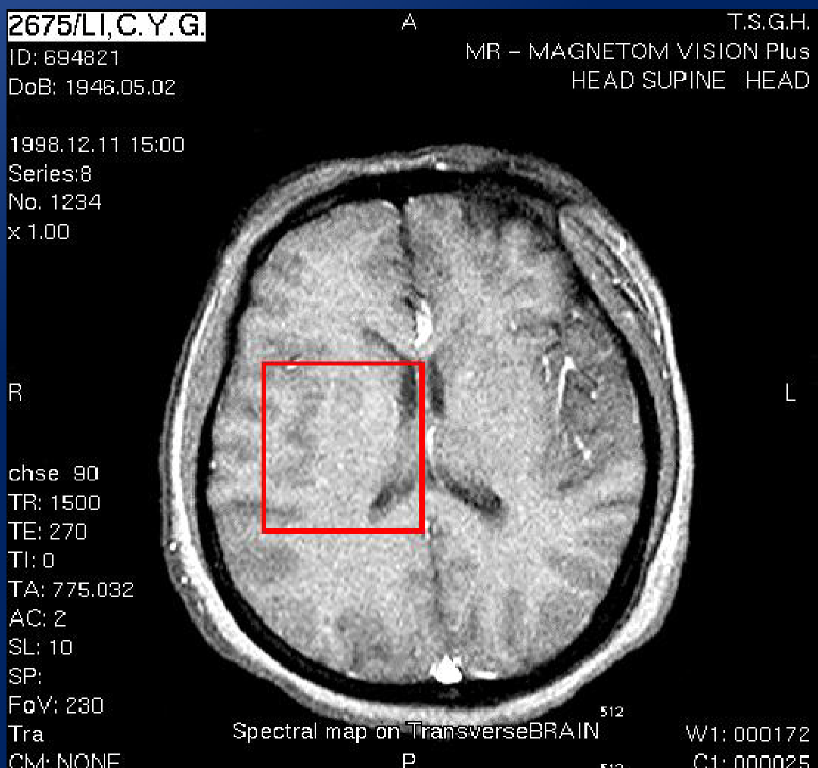
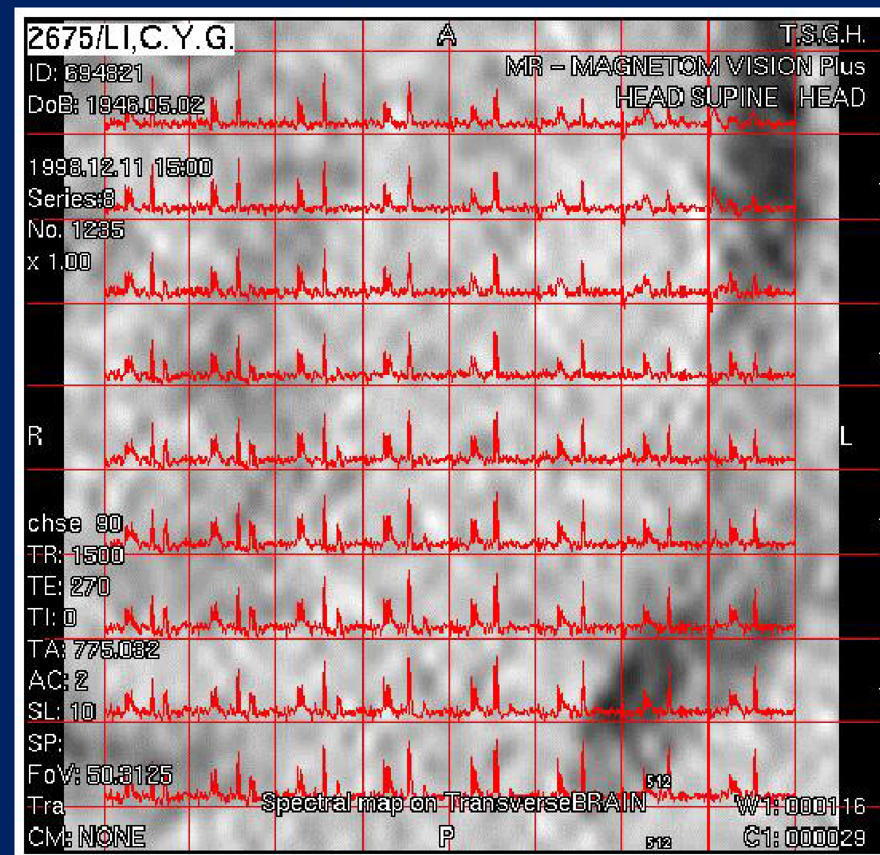
9%
0%
0
20
40
60
80
OMP +
Density Penalty

Other CS applications: MRSI

- Mapping Metabolites in Human Brain





- T1W

Regional Spectra

Compressed sensing for resolution enhancement of hyperpolarized ^{13}C flyback 3D-MRSI

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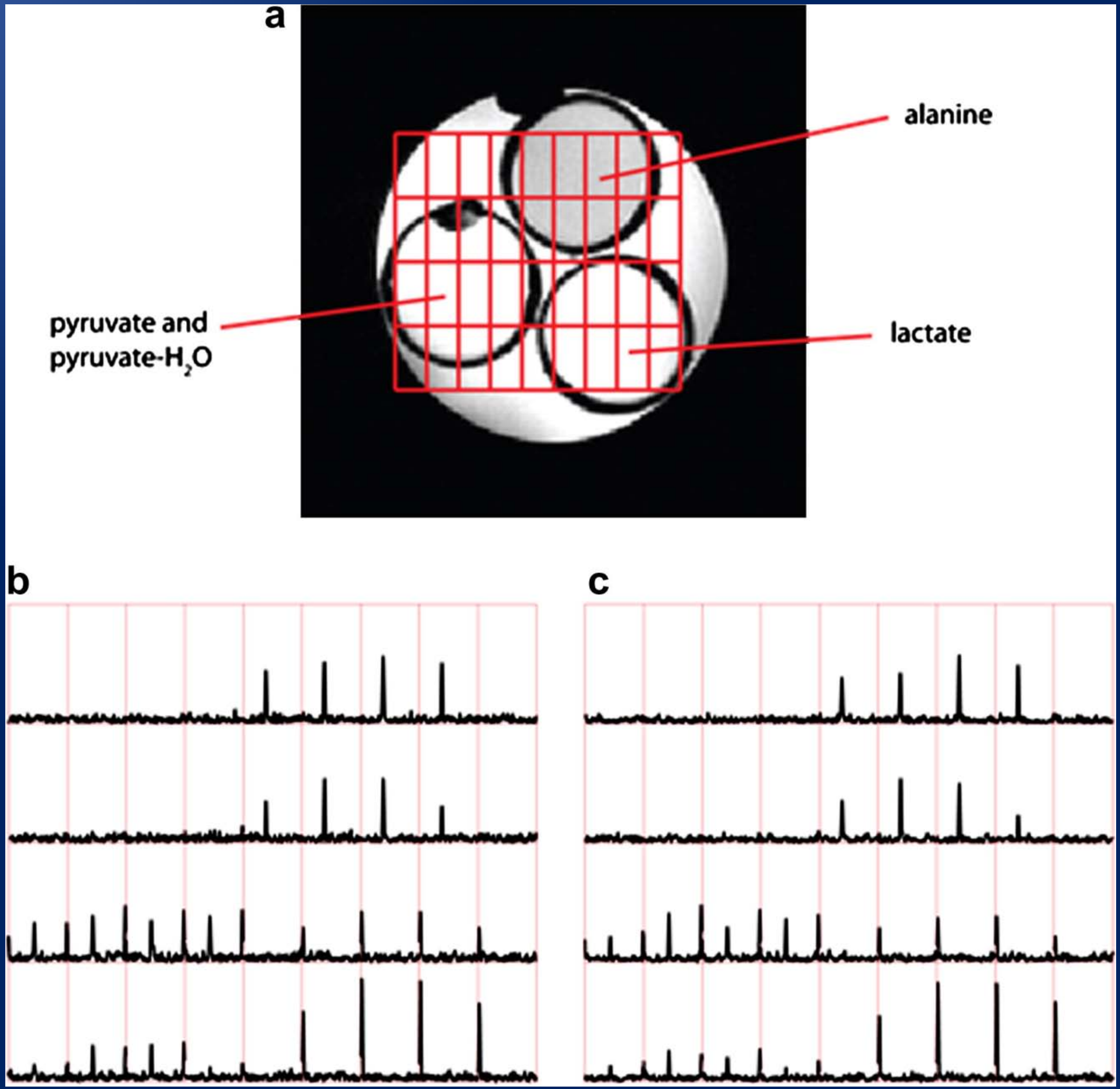
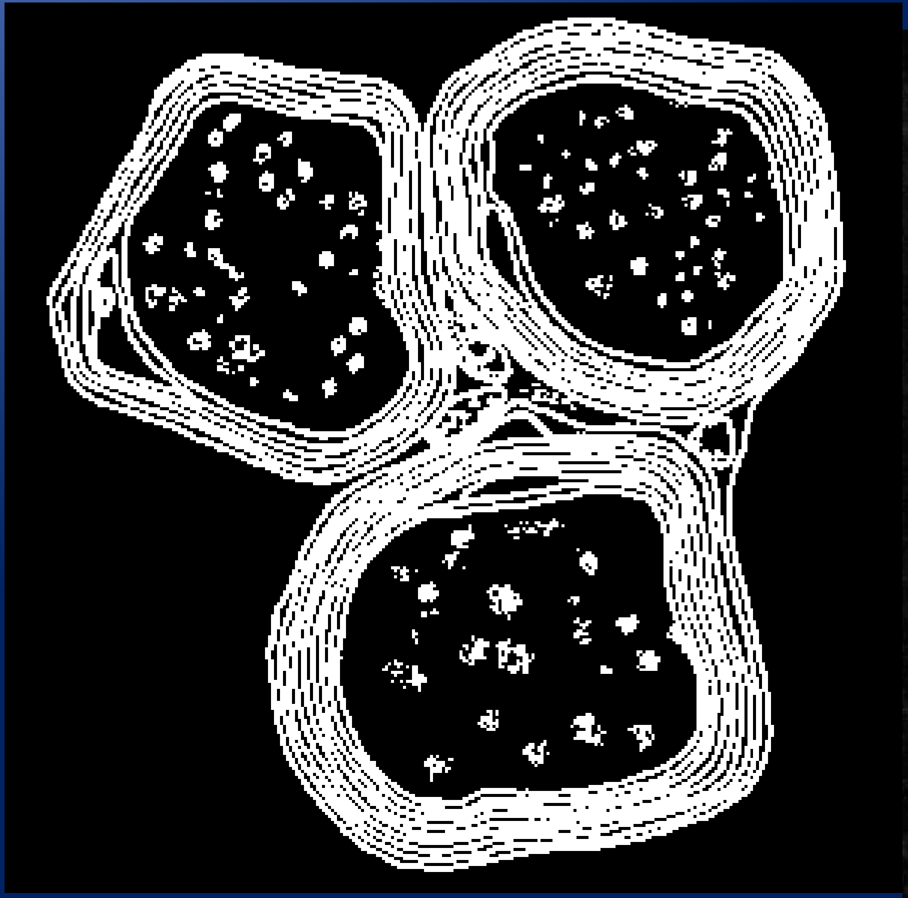
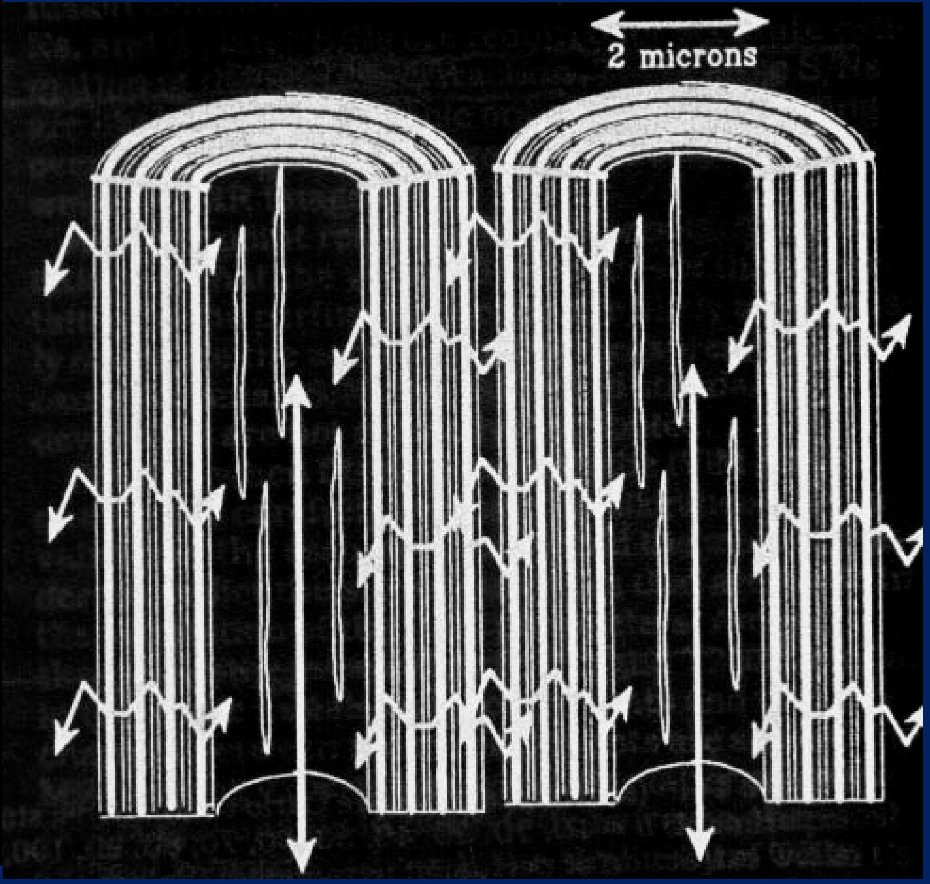
Non-accelerated Reference



3x acceleration

Other CS applications: HARDI & Fiber tracts

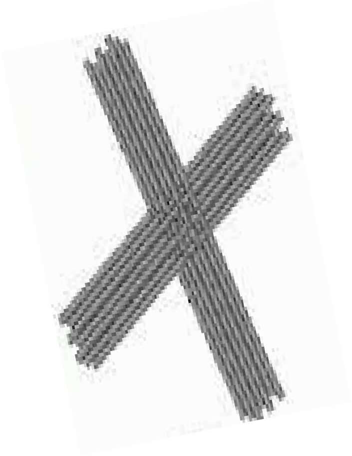




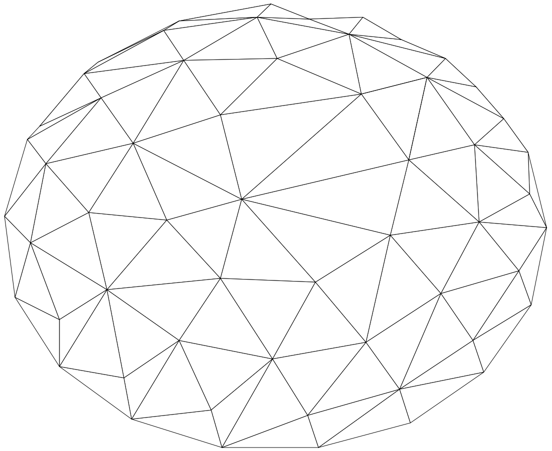
- Typical Water Diffusion Barrier

Other CS applications: HARDI

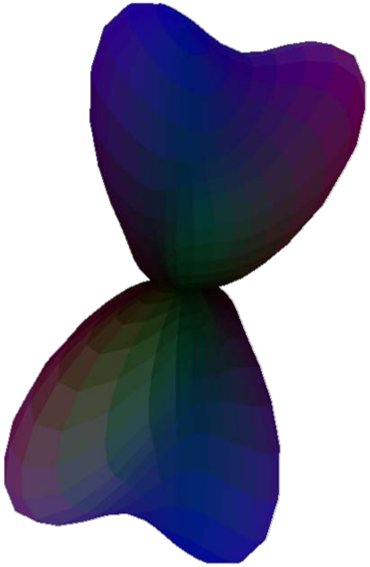
- HARDI encodes ADC on several direction to resolve complex microstructure



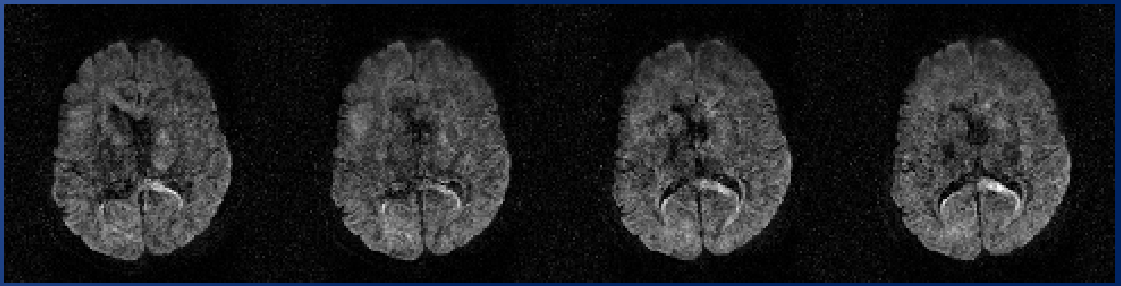
Diffusion Encoding Directions



Diffusion Profile



Resolving the Orientation: CFARI



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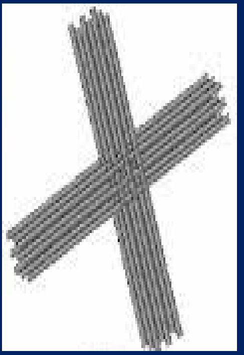
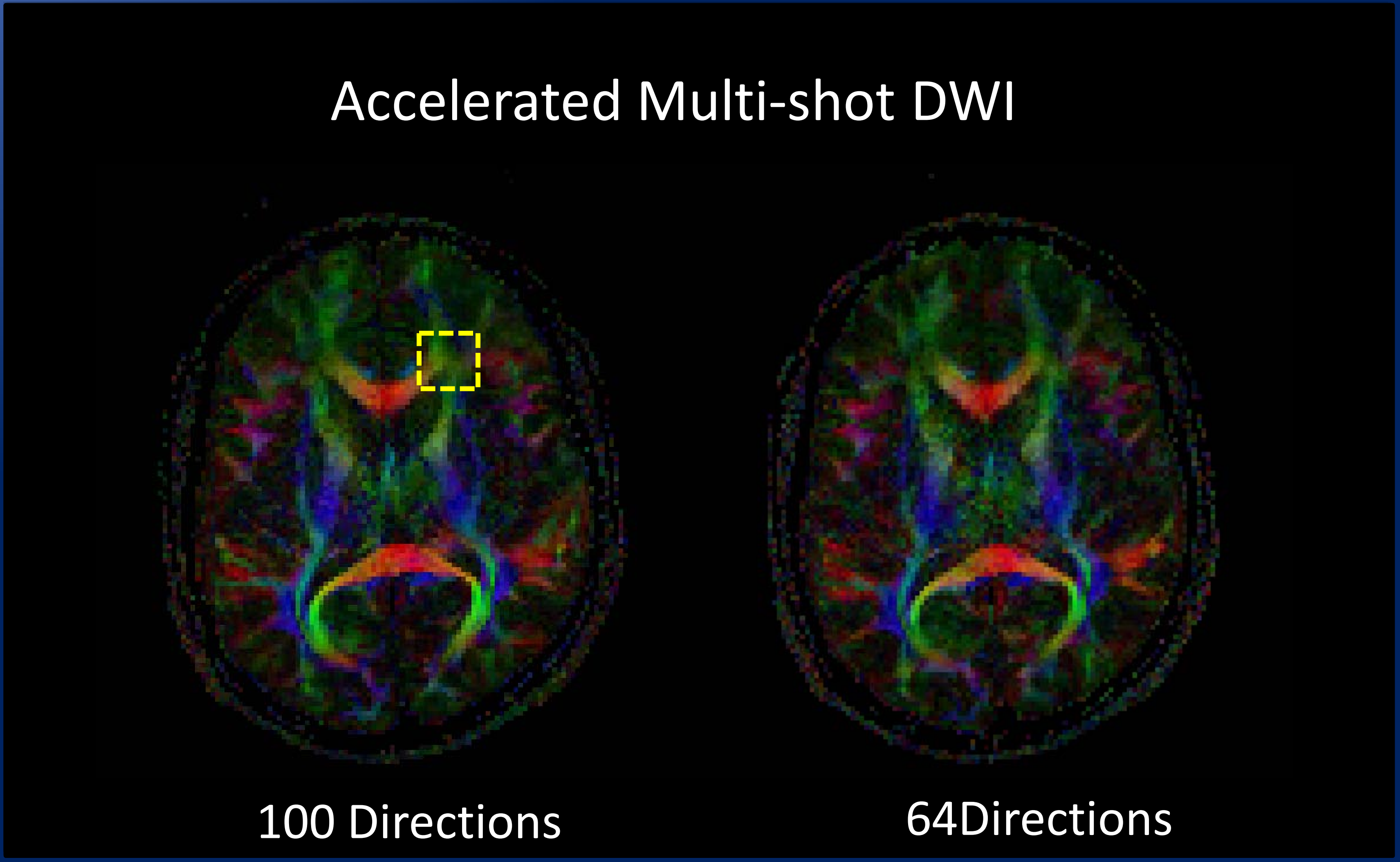


$$\hat{O}(y,d) = S_0 \sum_i f_i e^{-\left(b\cdot\vec{d}^T D_i \vec{d}\right)}$$

$$f = min_{f:f_i\in[0,\infty)} \left\| \sum_i f_i e^{-\left(b\cdot\vec{d}^T D_i \vec{d}\right)} - \hat{O}(y,d)/S_0 \right\|_2^2 + \beta\|f\|_1$$

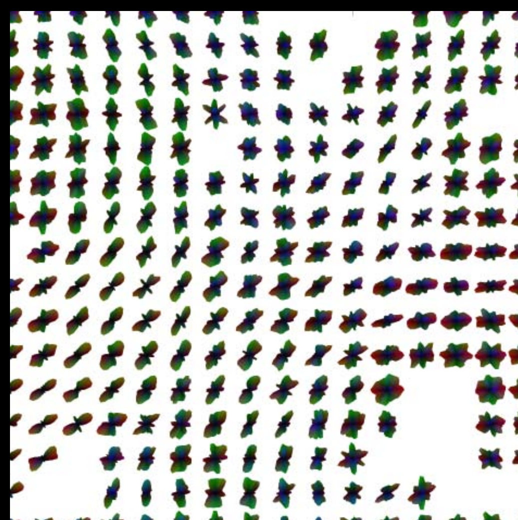
D: Modelled prolate diffusion tensor
f_i: Components of each modeled profile

Result : Color FA map

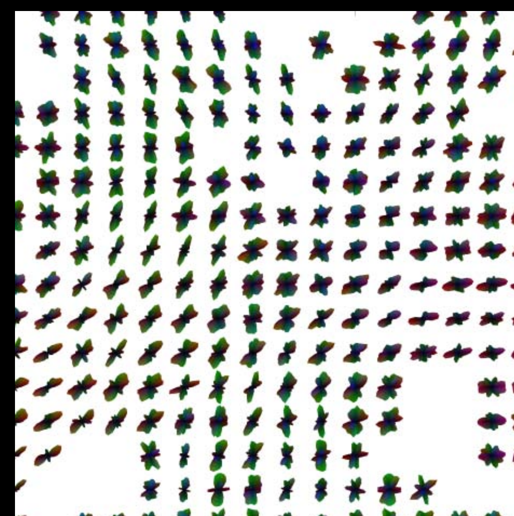


Orientation Map

Accelerated Multi-shot DWI



100 Directions



64Directions

Something About CS

- A very powerful tool
  - Not only useful in MRI but other Multimedia
- Sparsity is the key toward faster MR scan
  - Fast super resolution imaging
- Reconstruction may take much time.

高等磁共振影像技術

動態加速影像與壓縮感知
Accelerated MRI & Compressed Sensing

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